## SUMMARY



Non-linear calculations of composite sections and semicontinuous joints

Nicht-lineare Berechnungen von Verbundquerschnitten und biegeweichen Knoten

## continuous joints

Non-linear calculations of composite sections and semi-

First of all the Eurocodes opened the door to non-linear calculations in view of elastic-plastic material laws and geometrical non-linearities. Using the structures up to their plastic capacity instead of the conventional elastic design philosophy enables a drastic reduction of material use at least in view of the ultimate limit state. Resulting from this reduction of construction dimensions automatically the requirements at serviceability limit state are often governing the design with the effect of increasing demands on sophisticated non-linear design tools.

In the recent decade the combined use of different materials within construction elements or even the mixture of construction methods within one structure became more and more disseminated due to the remarkable advantages in view of economy and construction time. In modern construction different materials are used consciously at those locations within the structure where they develop their specific advantages. The most frequently applied combination of construction materials for buildings as well as bridges is that of structural steel and concrete. The importance of such steel-concrete composite sections is stressed by the existence of an own Eurocode dedicated to this building technology.

A quite new approach is the reality-related representation of joints in the analysis of structural systems. Joints in a wider sense are connecting all kinds of non-monolithic structural elements and are therefore the crucial key points within members themselves as well as within the overall structure. Nevertheless due to a lack of more realistic knowledge on their characteristics and the way of modelling, the joints so far have been neglected badly and simplifying assumptions concerning their representation often led to time-consuming and expensive constructions. This work focuses on the joints between beam and column members with special regard to their non-linear characterisation. With the help of an analytical component approach the so-called semi-continuous joint response can be determined. This method has successfully been applied to steel and composite structural joints, however it could easily be enlarged to other materials.

The first chapter of this doctoral thesis - the introduction - contains general considerations about new building concepts and different construction methods followed by a detailed presentation of construction elements in so-called mixed structures. This covers the different types of slabs, beams, columns and - as new construction elements influencing the design calculations - the joints. To avoid misunderstandings it continues with the definitions and the terminology
used in composite and mixed construction. Special attention is paid to the differences between bare composite construction and the more general (sometimes even already unconsciously used) mixed building technology. From the shear-slip characteristic of a single connector it is led over to the behaviour of the overall shear joint between the slab and the sheeting and the slab and the beam. The joint's resistance is linked to the degree of shear connection, the so-called shear interaction concerns the stiffness and the third key value is the joint's deformation capacity. The erection sequence determines the type of composite action.

The second chapter deals with composite floors consisting of the slabs and the beams. In the general part the principles of composite action between the contributing elements are discussed. A further chapter is dedicated to the new composite slim-floor systems, where the steel beams are already integrated into the slabs with the result of a drastic reduction of construction height. After some construction principles the main advantages of this innovative technology are presented. The enormous flexibility is emphasised giving some examples in view of slab types, floor beams, columns and joints. A special testing programme has been worked out for the characterisation of a new dowel system, the so-called punched shear connector. First of all the principle idea and the manufacturing process of these dowels which are already integrated into the floor beam are described. For the testing programme it is distinguished between fundamental push-out tests and final beam tests, which have been carried out for the validation of the gained results. The test evaluation procedure is described in detail for the design resistance, the stiffness and the important deformation capacity applying the method of partial shear connection.

A real focal point of this thesis is the theoretical background of joint representation described in chapter 3. Comprehensive knowledge on this field has already available at the Institute of Steel and Timber Construction in Innsbruck. Basing on that this chapter has developed iteratively as a result of numerous proposals, which have been introduced by the author into the Drafting Group for Composite Connections, aiming at the provision of Model Clauses for ENV 1994-1-1. First general considerations are made about the new semi-continuous joint philosophy, followed by the description of the component method as an analytical method of joint characterisation. Following the explanation of the different available component models, the basic joint components are listed. Refinements in view of the tension region of composite joints has become necessary and are therefore is treated in a separate chapter. Untiring discussions within international committees about component characterisation on the basis of test results
have shown the importance of a unified test evaluation concept, which is given herein for all the key characteristics: initial stiffness, design resistance, elastic limit and rotation or deformation capacity. The assembly of contributing nonlinear components to the overall joint response is one of the main tasks of this doctoral thesis. For that purpose an own computer program has been realised details are described in chapter four, however already chapter three gives most of the theoretical background in view of translational and rotational grouping of component springs. Further comparisons are made between sophisticated and simplified assembly models. Passing different possibilities of joint modelling a lot of background considerations are made in view of the differences between joint modelling reflecting the actual behaviour and simplified joint modelling (as used in the Eurocodes). A so-called joint transformation is developed to improve accuracy when applying the simplified joint model. This is followed by comments on the joint classification concept introduced by the Eurocodes. For a first comparative judgement of joint test results a joint assessment system is proposed leading to a simple possibility to define joint families. The chapter is concluded with final guidelines on joint representation within the global frame analysis as a summary of the previous considerations.

After these theoretical and quite sophisticated investigations their practical applicability is proved by a specially developed computer program called CoBeJo (Composite Beams and Joints). This program is dedicated to scientists as well as innovative practitioners covering the already described three fields of modern engineering: non-linear calculations of composite sections and semi-continuous joints. Special attention is paid to an advanced and self-explaining postprocessing. A lot of graphical tools are provided to inspect the various results. At present two main modules for the separate non-linear characterisation of composite sections and semi-continuous joints are available. An extension to a common action of beam and joint elements within overall semi-continuous beams and frames is planned.

At the beginning the mechanical background as well as the handling of the first module covering composite but also steel beam cross sections (BCS) is described starting with the definition of the section geometry and the respective material laws and ending with all possible calculations for the steel section alone, the concrete section alone and the overall composite section. The following non-linearities are taken into consideration: non-linear material laws of steel, reinforcing steel and concrete including the tension stiffening effect of concrete between the cracks. Furthermore a jump of the strain distribution at the layer of
shear connection between the steel section and the reinforced concrete section due to incomplete shear interaction may be considered.

The second part of the CoBeJo description is dedicated to the module of joint characterisation (JOINT). In the introduction the main principles of modern joint representation are repeated, maybe giving a better general overview of this new philosophy to a newcomer than the more sophisticated chapter on joint representation itself. After the explanation how a joint configuration can be defined by giving the characteristics of all contributing translational components, the iterative assembly procedure is described in detail followed by numerous graphical tools helping to explore the joint response in a quite convenient way. So on the one hand this module provides a powerful tool for scientific purposes in view of further parameter studies and on the other hand it is meant to support the promotion of the semi-continuous joint concept.

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## Nicht-lineare Berechnungen von Verbundquerschnitten und biegeweichen Knoten

Zwei Bereiche moderner Ingenieurbautechnologie waren Thema dieser Doktorarbeit: Stahl-Beton-Verbundquerschnitte für Deckenträger, Deckenplatten und Stützen und Riegel-Stützen-Verbindungen (sogenannte Knoten).

Das erste Kapitel - die Einleitung - beinhaltet allgemeine Überlegungen zu neuen Bauwerkskonzepten und verschiedenen Baumethoden, gefolgt von einer detaillierten Beschreibung von Bauteilen der sogenannten Mischbautechnologie. Dabei werden verschiedene Typen von Deckenplatten, Deckenträgern, Stützen und - als neue Bauteile, die die Tragwerksbemessung beeinflussen - Knoten behandelt. Um Mißverständnissen vorzubeugen, folgen Definitionen und Begriffsbestimmungen für die Verbund- und Mischbauweise. Dabei wird speziell auf die Unterschiede zwischen reinem Verbundbau und der allgemeineren (ev. sogar schon unbewußt eingesetzten) Mischbautechnologie eingegangen. Aus dem Ver-bund-Schlupf-Verhalten einzelner Verbindungsmittel wird danach das Verhalten der gesamten Dübelfuge zwischen Betonplatte und Stahlblech bzw. zwischen Deckenplatte und Deckenträger hergeleitet. Die Tragfähigkeit der Fuge wird als Verdübelungsgrad angegeben, die Verbundnachgiebigkeit hängt mit deren Steifigkeit zusammen, und als dritter Kennwert ist das Verformungsvermögen zu nennen. Der Bauablauf wiederum bestimmt die Verbundart.

Im zweiten Kapitel geht es um Verbunddecken, bestehend aus Deckenplatten und Deckenträgern. Zuerst werden die Grundlagen der Verbundwirkung zwischen den einzelnen Bauelementen im allgemeinen Teil erörtert. Ein weiteres Kapitel ist den Flachdecken (slim-floors) gewidmet, wo der stählerne Deckenträger in die Deckenplatte integriert wird und dadurch bedeutend an Bauhöhe eingespart werden kann. Im Anschluß an einige Konstruktionsprinzipien werden die Vorteile dieser innovativen Baumethode vorgestellt. Deren beeindruckende Flexibilität wird dabei anhand von Beispielen hinsichtlich Platten-, Träger-, Stützen- und Knotentypen unterstrichen. Zur Ermittlung des Verhaltens eines neuen Verbindungsmittels, des sogenannten Stanzdübels, wurde ein spezielles Versuchsprogramm ausgearbeitet. Zuerst werden die Grundlagen sowie die Herstellung dieser Dübel beschrieben, die bereits direkt aus dem Trägermaterial geformt werden. Beim

Versuchsprogramm wird zwischen grundlegenden Push-out Tests und abschlieBenden Trägerversuchen, die lediglich zur Bestätigung der zuvor gewonnenen Ergebnisse dienten, unterschieden. Die Testauswertung wird detailliert für die Bemessungstragfähigkeit, die Steifigkeit und die wichtige Verformungskapazität erläutert, wobei die Methode des nachgiebigen Trägerverbundes zur Anwendung kam.

Einer der Hauptschwerpunkt dieser Arbeit ist der theoretische Hintergrund der Modellbildung von Knoten, der in Kapitel 3 beschrieben wird. Umfassendes Wissen auf diesem Gebiet war bereits zuvor am Institut für Stahlbau und Holzbau in Innsbruck verfügbar. Darauf aufbauend entstand dieses Kapitel sukzessive aus zahlreichen Vorschlägen, die der Autor in die Drafting Group for Composite Connections - ein Gremium zur Erstellung von Model Clauses für die ENV 1994-1-1 eingebracht hat. Nach allgemeinen Überlegungen zu biegeweichen Knoten folgt die Beschreibung der Komponentenmethode, einer analytischen Vorgangsweise zur Bestimmung des Knotenverhaltens. Zuerst werden verfügbare Komponentenmodelle vorgestellt, danach sind zahlreiche Knotenkomponenten zusammengefaßt. Die erforderlich gewordenen Verbesserungen der Modellierung der Zugregion in Verbundknoten werden detailliert in einem eigenen Kapitel beschrieben. Unermüdliche Diskussionen innerhalb internationaler Forscherkreise haben die Notwendigkeit eines vereinheitlichten Testauswerteverfahrens für die Komponentencharakterisierung gezeigt. Hierin wird ein Vorschlag für folgende Kennwerte unterbreitet: Anfangssteifigkeit, Bemessungstragfähigkeit, elastische Grenztragfähigkeit und Rotations- bzw. Deformationsvermögen. Die Gruppierung (Assembly) aller am Knotenverhalten beteiligten nichtlinearen Komponenten zum Gesamtverhalten des Knotens ist ein weiterer Hauptschwerpunkt dieser Dissertation. Zu diesem Zweck wurde ein eigenes Computerprogramm entwickelt. Details dazu werden zwar erst in Kapitel 4 beschrieben, jedoch gibt bereits dieses Kapitel Einblick in die theoretischen Hintergründe der Weg- und Drehfedergruppierung von Komponentenfedern. Darüber hinaus werden Vergleiche zwischen wissenschaftlichen und vereinfachten Assembly-Modellen angestellt. Nach einer allgemeinen Vorstellung der verschiedenen Möglichkeiten der Knotenmodellierung werden zahlreiche vergleichende Studien zwischen dem realitätsnahen und dem
vereinfachten Knotenmodell, wie es in den Eurocodes angegeben ist, durchgeführt. Eine sogenannte Knotentransformation wurde entwickelt, um die Ergebnisse bei Anwendung des vereinfachten Knotenmodells zu verbessern. Es folgen Kommentare zum Eurocode-Konzept der Knotenklassifikation. Zum Zweck einer Erstbeurteilung der Ergebnisse aus Knotenversuchen wird ein AssessmentVerfahren vorgeschlagen. Dies ermöglicht eine einfache Bestimmung von gleichartigen Knotenfamilien. Abschließend werden die vorhergehenden Erkenntnisse in Form von Richtlinien für die Modellbildung von Knoten bei der Tragwerksberechnung zusammengefaßt.

Nach diesen theoretischen Untersuchungen wird deren praktische Umsetzbarkeit anhand eines speziell für diesen Zweck entwickelten Computerprogrammes namens CoBeJo (Composite Beams and Joints) unter Beweis gestellt. Dieses Programm ist sowohl für Forscher als auch innovative Praktiker bestimmt und umfaßt die bereits zuvor erwähnten Bereiche moderner Bautechnologie: nichtlineare Berechnung von Verbundquerschnitten und biegeweichen Knoten. Großer Wert wurde dabei auf ein fortschrittliches und selbsterklärendes Post-Processing gelegt. Eine Vielzahl von Grafikwerkzeugen ermöglichen ein detailliertes und übersichtliches Studium der Ergebnisse. Derzeit sind zwei wesentliche Module für die getrennte nichtlineare Charakterisierung von Verbundquerschnitten und biegeweichen Knoten verfügbar. Eine Erweiterung auf ein gemeinsames Zusammenwirken von Träger- und Knotenelementen innerhalb eines biegeweichen Trägers oder Rahmens ist geplant.

Zu Beginn wird der mechanische Hintergrund sowie die Handhabung des ersten Moduls beschrieben, der zur Berechnung von Stahl- und Verbundquerschnitten ( $B C S$-Beam Cross Section) dient. Im Anschluß an die Eingabe der Quer-schnitts- und Materialkennwerte sind die Berechnungsverfahren für den isolierten Stahlquerschnittsanteil, den isolierten Betonquerschnittsanteil und den GesamtVerbundquerschnitt erläutert. Folgende Nichtlinearitäten werden dabei berücksichtigt: das nichtlineare Stoffgesetz von Baustahl, Bewehrungsstahl und Beton unter Berücksichtigung des „Tension stiffening effects" zwischen den Rissen, sowie ein Dehnungssprung in Höhe der Dübelfuge zwischen dem Stahl- und dem bewehrten Betonanteil infolge der Dübelnachgiebigkeit.

Der zweite Teil der CoBeJo Beschreibung behandelt das Modul zur Knotencharakterisierung (JOINT). In der Einleitung werden die Grundlagen moderner Knotenmodellbildung in einer praxisorientierteren Weise als in Kapitel 3 wiederholt, um einem Newcomer auf diesem Gebiet einen besseren Überblick zu geben. Die Eingabe der zu untersuchenden Knotenkonfiguration erfolgt über die Definition des Verhaltens der jeweiligen Wegfeder-Komponenten. Danach wir die iterative Assembly von den einzelnen Wegfedern zur Knotendrehfeder im Detail beschrieben, gefolgt von den zahlreichen Grafikwerkzeugen, die das Erforschen des Knotenverhaltens auf eine komfortable und spielerische Weise ermöglichen. Infolgedessen stellt dieses Modul einerseits ein leistungsfähiges Werkzeug für wissenschaftliche Zwecke hinsichtlich weiterer Parameterstudien dar, andererseits ist es auch dazu bestimmt das recht neue Konzept von biegeweichen Knoten in der Praxis zu etablieren.

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# NON-LINEAR CALCULATIONS <br> OF COMPOSITE SECTIONS AND SEMI-CONTINUOUS JOINTS 

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## 4 Computer Program

### 4.1 General

Within the scope of this work two of the main modules of a software-package called "CoBeJo" (Non-linear Calculations for Composite Beams and Joints) have been worked out. The final aim of this program will be the calculation of composite beams with partial shear connection and therefore also incomplete shear interaction taking into consideration the material non-linearities, the non-linear force-deformation behaviour of the shear interface and furthermore the non-linear moment-rotation response of the composite joints. Out of this task a clear subdivision into the following basic jobs arose:

- input and characterisation of the beam's cross-section (BCS)
- input and characterisation of the joint configuration (JOINT)
- global analysis of the entire beam formed by finite beam elements and joints (planned for the future)

Each of these three jobs is solved by a separate computer program representing the basic modules of the CoBeJo package. That results in clear input and output windows, where the user will not get lost due to the complexity of the job. The calculation is solved step by step with a multitude of provisional results, which at one hand ease checks and at the other hand enable to adjust the analysed system interactively in view of economical optimisation. Besides especially for scientific purposes parameter studies have successfully been performed. In addition the results of the single modules can also be used as input data for other programs using the output files as interface.


Fig.4-1 Structure of the computer program package CoBeJo

As the acceptance of a computer program mainly depends on the simplicity and clearness of the pre- and post-processing the object-orientated MS Windows-compiler Visual Basic has been chosen. In the following the theoretical background as well as the handling of the realised two main modules „Beam Cross Section" and „Beam-toColumn Joint" will be described in detail.

Remark: A dot (not a comma) has to be defined as decimal separator in MS Windows for all modules of this program.

### 4.2 Beam Cross Section

### 4.2.1 General

First of all a comfortable procedure for the input of composite beam sections should be provided followed by calculations to study the member response. The clear separation between the steel and the reinforced concrete section part has strictly been realised throughout the whole program for the input as well as for the results. To make accessible the influences of the section parts on the entire composite member response the behaviour of the steel and concrete part at one hand can be analysed separately and at the other hand many of the composite results are given as the sum of steel and concrete. Again this eases the check of the results and also enables to examine, if both section parts, steel and reinforced concrete, are used in an efficient way or if failure of the overall composite member is dominated by one section part while the other one is only hardly stressed.

The section analysis is based on the assumption of a linear strain distribution, taking into consideration a jump of strains at the shear interface due to the incomplete interaction. Generally a constant distribution of strains in transverse direction is expected. To take account for that effective concrete widths have to be used. From the given stress-strain diagrams of the section elements the stress distribution can be derived from which the internal forces and moments, both for the section parts and for the entire composite section, can be calculated. Investigating the whole range of possible strain distributions interactions between internal forces of the entire section and that of the section parts as well as coherence between internal forces and deformations can be gained.


Fig.4-2 Structure of the module „Beam Cross Section"

At the beginning only the fully plastic limit state has been treated assuming a perfectly plastic behaviour of each element. In a second step the program has been extended to the whole elasto-plastic range. At one hand self-evidently the input of the material characteristics had to be extended and at the other hand the iterative calculations had to be fully changed. A special aim was to bring in the tension stiffening effect of the concrete between the cracks following the guidelines given in the new Composite Bridge Code [62] and to enable partial shear connection also in the hogging moment regions beyond the rules of ENV 1994-1 [59,109]. As the elasto-plastic calculation of course also covers the fully plastic state as an extreme limit the module for plastic section analysis will not be described herein any more. It only proved to be very useful for checks of the elasto-plastic module as both programs are based on completely different calculation methods.

Summarising the module for the Beam Cross Section represents an instrument for studying the response of composite members within the full elasto-plastic range considering the contribution of the individual section parts steel and concrete. The principle structure of this module can be seen in Fig.4-2.

### 4.2.2 Section management



Fig.4-3 Start-up window of the module „Beam Cross Section" (profile.frm)

Fig.4-3 shows the start-up window of the module "Beam Cross Section". It serves for the file-management of composite sections and gives the link to the windows for the input and calculation of the steel and the concrete section part as well as to the composite calculations.

A special composite cross section consisting of a hat-shaped steel beam and an upper reinforced concrete plate has been chosen for the following examples. The cross sectional input data, which can be printed with the button "Print", is given on page 157. All section data including section name, safety factors, geometry and material properties are saved in a file with the default-extension "*.pro". The input file of the example section "diss.pro" can be seen on page 158. Clicking on the "Load section" or "Save section" buttons activates the window for the file management (Fig.4-4). Apart from the possibility to load or save section data buttons are provided for a small variety of disk manipulation like „Create Directory", „Remove Directory", „Rename File", „Copy File" and "Delete File".


Fig.4-4 File management (file.frm)

It was tried to keep the program self-explaining, however the possible further operations concerning the start-up window should also be listed: A special text field is reserved for a section name, the button "New section" deletes the actual profile, a section can be rotated $90^{\circ}$ or $180^{\circ}$ and the field for mirror options enables to reduce the input procedure for symmetrical sections (remark: the reinforced concrete section will only be mirrored horizontally and not vertically).


Version: 0.1 b

## Input file: diss.pro

elasto-plastic (input-file saved by plastic or elasto-plastic module)
section ( $0=$ user, $1=$ hat, $2=1$ ), $1 \quad$ (section type)
hat-shaped beam (name of section)
hogging mom., 0 (only necessary for plastic module)
safety factor steel, 1.1
safety factor, stress-factor(EC4) concrete, 1.5, 0.85
safety factor reinforcement, 1.15
beta-tension-stiffening, 0.4
fcteff/fctm, 1
symm.conditions (-1,0,1 hor.left/no/right, $-1,0,1$ ver.steel down/no/up), 1, 0
elementlist steel

| $\begin{aligned} & \text { El.no. } \\ & 1 \text { to } 40 \end{aligned}$ | type ( $0=r e c t$. | (left corner) [mm] | (lower corner) [mm] | width [mm] | height [mm] | yield strength <br> [ $\mathrm{N} / \mathrm{mm}^{2}$ ] | E-Modulus [ $\mathrm{kN} / \mathrm{mm}^{2}$ ] |  | ultimate strain [1] | flange $=0$ web=1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 50 | 0 | 250 | 15 | 355 | 210 | 510 | 0.1 | 0 |
| 2 | 0 | 175 | 15 | 10 | 200 | 380 | 210 | 450 | 0.1 | 1 |
| 3 | 0 | 185 | 205 | 45 | 10 | 380 | 210 | 450 | 0.1 | 0 |
| 4 | . | - | . | - | . | - |  | - |  | - |
| 5 | - | - | - | - | - | - |  |  |  | $\cdot$ |
| 6 | - | - | - | - | - | - |  | - | - | $\cdot$ |
| 7 | - | - | $\cdot$ | - | - | - |  |  | - | - |
| 8 | - | - | - | - | - | - | - | - | - | $\cdot$ |
| 9 | - | - | - | - | - | - |  | - | - | - |
| 10 | - | - | - | - | - |  |  |  | - | - |
| 11 | 0 | 300 | 0 | 250 | 15 | 355 | 210 | 510 | 0.1 | 0 |
| 12 | 0 | 415 | 15 | 10 | 200 | 380 | 210 | 450 | 0.1 | 1 |
| 13 | 0 | 370 | 205 | 45 | 10 | 380 | 210 | 450 | 0.1 | 0 |
| 14 | - |  | - | - | - | - |  |  |  | - |
| 15 | - | - | - | - | - | - |  | - |  | $\cdot$ |
| 16 | $\cdot$ | - | - | - | - |  |  | - |  | - |
| 17 | - | - | - | - | - | - | - | - | $\bigcirc$ | - |
| 18 | $\cdot$ | - | - | - | $\cdot$ | - |  | - |  | $\cdot$ |
| 19 | $\cdot$ | $\cdot$ | $\cdot$ | - | $\cdots$ | , |  | - | S | - |
| 20 | $\cdots$ | $\cdots$ | : | - | - | $\cdots$ |  | - |  | - |
| 21 | - | - | - | - |  | - |  | - |  | $\cdots$ |
| 22 | $:$ | $:$ | - | - | - |  |  | - |  | - |
| 23 | - | - | - | $:$ | : |  |  | - | - | : |
| 24 25 | . | $:$ | : | - | - | - | - | $\cdots$ | - | - |
| 26 | - | - | $\cdot$ | $\cdot$ | . |  | - |  | - | - |
| 27 | - | - | - | - |  |  | - |  | - | - |
| 28 | - | - | - | - |  |  | - |  | - | $\cdot$ |
| 29 | - | - | - | - |  |  | $\cdot$ | - | - | - |
| 30 | - | $\cdot$ | - | - | - |  | - |  | - | $\cdot$ |
| 31 | - | $\cdot$ | - | - | . | - | - | - | - | - |
| 32 | - | - | - | - | . |  | $\cdot$ |  | - | $\cdot$ |
| 33 | - | - | - | - | - |  | - | - | - | - |
| 34 | $\cdot$ | $\cdot$ | - | - | . | . | - | - | - | $\cdot$ |
| 35 | - | - | - | - |  |  | - | - | - | - |
| 36 | $\cdot$ | - | - | $\cdot$ |  | , |  | - | - | - |
| 37 | - | - | - | - |  |  |  | - | - | - |
| 38 | $\cdot$ | - | - | - |  | - |  | - | - | - |
| 39 | - | - | - | - | - | - |  | - | - | - |
| 40 | - | - | - | - | $\cdot$ | $\cdot$ |  | $\cdot$ | - | - |
| elementlist | oncrete |  |  |  |  |  |  |  |  |  |

elementlist concrete
shear


A special quick-input procedure has been arranged for the often-used composite sections with I-shaped (Fig.4-5) or hat-shaped (Fig.4-6) steel beams. If desired the gained section from the quick-input then can be modified in detail within the steel and concrete input windows.


Fig.4-5 Quick-input for l-shaped steel beams (i_ prof.frm)


Fig.4-6 Quick-input for hat-shaped steel beams (hutprof.frm)

For I-shaped steel sections default values for IPE, HEA and HEB beams can directly be taken from the file ,,iprofile.dat" in the following format: Name, height, width, thickness of web, thickness of flange, fillet radius. As the section has to be built up only by rectangular elements the fillet radius is approximated as shown in Fig.4-7.


Fig.4-7 Approximation of fillet radius by rectangular elements

Finally it should not be forgotten to mention the precision factor, which is set equal to one in the start-up window. Generally all accuracy pre-settings within the iteration loops have been defined as a compromise between precision and calculation time. In the summary of this chapter the convergence criteria defined for each iteration loop are given. Further for each iteration result the final deviation will be declared. So if this indicated deviation should exceed the desired limits the accuracy can be improved by raising the precision factor. However thanks to the quick computers in most cases even the wise absolute lower accuracy limits (e.g. 1 N as absolute lower precision limit for the inner shear force) are already decisive using the pre-set precision factor 1.0.

The button „STEEL" will close the start-up window and open that for the steel section part, which will be described in detail in the following chapter.

### 4.2.3 Steel section

### 4.2.3.1 Section input

The steel section window, shown in Fig.4-8, first of all enables the input of the steel section part consisting of up to 10 rectangular elements. In view of the possibility to mirror the section horizontally and vertically the final steel section so may consist of up to 40 elements. The 10 basic elements are marked with different colours. To point out the mirrored elements they are drawn with a more pale colour, what is also stressed in the "Mirror"-box. For each element the coordinates of one corner ( $\mathrm{x}, \mathrm{y}$ ) and the width and height ( $\mathrm{b}, \mathrm{h}$ ) to the other corner can be defined. Also negative values are accepted, however then the $x$ - and $y$-coordinates will automatically be re-calculated for the lower left corner. To facilitate the input the coordinates of the actual mouse-position are indicated at the top of the drawing. A single click within the drawing will write the coordinates of the nearest existing element corner to the $x$ - and $y$-boxes. A double-click is understood as the second corner of the actual element. Therefore the actual element width and height then are calculated as the distance between the given $x$ - and $y$-values and the nearest element corner from the double-click position. Attention has to be paid that the corners of mirrored elements cannot be caught; they will automatically be transformed to the basic element corners. Pressing „shift" while clicking on the drawing will define the height of a section coupling point (vertical section-to-section coupling for different cross section types within a beam), which will be used when placing element behind element for the global analysis.


Fig.4-8 Window for the input and calculation of the steel section part (prs.frm)

Further it can be chosen if an element participates in the vertical shear transfer as a part of the "web" or if there is no interaction between normal forces and shear forces when an element is defined as part of the „flanges". Web elements are indicated with a dashed border in the drawings. Their reduced stress-strain-diagram is calculated using the global procedure "querkraftsinteraktion" (see Fig.4-9, Fig.4-10). There the longitudinal yielding stress $f_{y k}$ fictitiously is reduced to $f_{y v}$ depending on the ratio between $\mathrm{V}_{\mathrm{Sd}}$ (actual interacting vertical shear force) and $\mathrm{V}_{\mathrm{Rd}}$ (plastic shear resistance) based on the Huber-Hencky-Mises-hypothesis. Fig.4-9 shows the difference between the applied theoretical approach of $\sigma$ - $\tau$-interaction and the one according to EC3 [58,108]. Simplifying only vertical web elements may be assumed to participate in shear.


Fig.4-9 Interaction between shear and longitudinal stresses

Apart from the geometrical properties also the material properties can be defined for each element separately. The constitutive law of a steel element is assumed to be symmetrical for tension and compression and is based on the following input data: Young modulus $E_{a}$, characteristic yield strength $f_{y k}$, ultimate strength $f_{u}$ and ultimate strain $\varepsilon_{u}$. Further it depends on the global partial safety factor of steel $\gamma_{\text {stee }}=\gamma_{\mathrm{a}}$ (default=1.0) and if it concerns a web element of course on the actual interacting vertical shear force $\mathrm{V}_{\text {Sd. }}$. It will automatically be checked that the actual shear force does not exceed the plastic shear capacity of all web elements together. Buckling will not be taken into consideration.

The definition of the stress-strain diagram for steel elements used in this program is summarised in Fig.4-10. The actual stress-strain diagram of each steel element can be seen true to scale when clicking on the „ $\sigma-\varepsilon^{\prime \prime}$-button (Fig.4-11). There will be the possibility to calculate the corresponding stress to a given strain and vice versa by a linear interpolation. Fig.4-12 gives the diagrams for the described example section using $\gamma_{a}=1.1$.

flange elements: $f_{y v}=f_{y k} \quad, \quad f_{u v}=f_{u}$ web elements: $V_{R d}=\sum_{\text {web elements }} b_{i} \cdot h_{i} \cdot \frac{f_{y k, i}}{\gamma_{a}} \cdot \frac{1}{\sqrt{3}}$ $V_{s d} \leq V_{\text {Rd }}: \quad f_{y v}=f_{y k} \cdot \rho \quad f_{u v}=f_{u} \cdot \rho$

$$
\rho=\sqrt{1-\left(\frac{V_{\mathrm{Sd}}}{\mathrm{~V}_{\mathrm{Rd}}}\right)^{2}}
$$

$$
\text { according to } f_{y v}^{2}+3 \tau^{2} \leq f_{y k}^{2}
$$

$\mathrm{V}_{\mathrm{Sd}}>\mathrm{V}_{\mathrm{Rd}}$ : beyond resistance

Fig.4-10 Fictitious stress-strain diagram of steel elements


Fig.4-11 Window for the actual stress-strain diagrams of steel elements (stoffga.frm)


Fig.4-12 Actual stress-strain diagrams for the example section

Before returning from the steel section window to the section management window or before starting any calculation it will be checked automatically if elements are overlapping and therefore these areas would be calculated more than once. Further after each element input the entire composite cross section automatically will be moved so that the minimum $x$-value of the entire section and the minimum $y$-value of the bare steel section (datum line) are equal to 0 .

Based on the so-defined steel section including geometry and material properties the following three calculations or iterations can be carried out:

- internal forces to a given strain distribution
- strain distribution to a given normal force
- N-M-relation for a defined curvature
4.2.3.2 Internal forces to a given strain distribution (steelstrain)


The actual linear strain distribution within the steel section can be defined by the strain at the steel bottom $(\mathrm{y}=0) \varepsilon_{0 \mathrm{a}}$ and the curvature $\kappa$ as shown in Fig.4-13.


Fig.4-13 Linear strain distribution

Based on that the global procedure "steelstrain" calculates the corresponding stresses of each steel element based on the material properties. If the stress at least in one point exceeds the design yield value ( $\mathrm{f}_{\mathrm{yv}} / \gamma_{\mathrm{a}}$ ) a non-elastic („ne") flag is set, otherwise elastic (.,el") will be indicated. An integration of the stresses over the whole steel section leads to the inner normal force $\mathrm{N}_{\mathrm{ia}}$ (tension positive). Integrating the stresses multiplied by their lever arms to the steel bottom line results in the moment $M_{a}{ }^{\prime}$, which of course is related to this datum line $y=0$ as described in [106] (sagging moments positive). Finally also the $y$-value of the neutral axis (N.A.a) is declared.

Fig.4-14 illustrates this basic calculation for the example section leading from strains to stresses and internal forces.

$$
\begin{array}{ccc}
\varepsilon_{0 \mathrm{a}}=0.569 \% \% \\
\mathrm{~K}=10 \% / \mathrm{m} \\
\mathrm{~V}=0 \mathrm{kN}
\end{array} \xrightarrow{\text { steelstrain }} \mathrm{M}_{\mathrm{a}}^{\prime}=139.4 \mathrm{kNm} . \begin{gathered}
\mathrm{N}_{\mathrm{ia}}=0 \mathrm{kN} \\
\text { elastic range } \\
\text { N.A. }=56.9 \mathrm{~mm}
\end{gathered}
$$



STEEL elements:

| el.no. | $\varepsilon_{\text {bottom }}[\%$ \% | $\varepsilon_{\text {top }}[\%]$ | $\sigma_{\text {bottom }}\left[\mathrm{N} / \mathrm{mm}^{2}\right]$ | $\sigma_{\text {top }}\left[\mathrm{N} / \mathrm{mm}^{2}\right]$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $\mathbf{0 . 5 6 9}$ | $\mathbf{0 . 4 1 9}$ | $\mathbf{1 1 9 . 4 4}$ | $\mathbf{8 7 . 9 4}$ |
| $\mathbf{2}$ | $\mathbf{0 . 4 1 9}$ | $\mathbf{- 1 . 5 8 1}$ | $\mathbf{8 7 . 4 4}$ | $\mathbf{- 3 3 2 . 0 6}$ |
| $\mathbf{3}$ | $\mathbf{- 1 . 4 8 1}$ | $\mathbf{- 1 . 5 8 1}$ | $\mathbf{- 3 1 1 . 0 6}$ | $\mathbf{- 3 3 2 . 0 6}$ |
| $\mathbf{e l a s t i c}$ |  |  |  |  |
| 11 | 0.569 | 0.419 | 119.44 | 87.94 |
| 12 | 0.419 | -1.581 | 87.94 | -332.06 |
| 13 | -1.481 | $\mathbf{- 1 . 5 8 1}$ | $\mathbf{- 3 1 1 . 0 6}$ | elastic |

Fig.4-14 Calculation example

### 4.2.3.3 Strain distribution to a given normal force (Nsteeliteration)

|  | 0 | $\begin{aligned} & {[\mathrm{kN}]} \\ & {[\text { [\% } \mathrm{w} / \mathrm{m}]} \end{aligned}$ | Iteration |
| :---: | :---: | :---: | :---: |
|  | 10 |  |  |
| precisio | ion | [kN] |  |

The global procedure „Nsteeliteration" goes the opposite direction iterating the corresponding strain distribution tor a given normal force $\mathrm{N}_{\mathrm{i} a}$ and a given curvature $\kappa$. This problem is solved in two steps:

First the ultimate boundaries of datum strains (at $y=0$ ) depending on the curvature $\kappa$ and the ultimate strain values $\varepsilon_{u}$ of the steel elements are derived within the procedure „steellimit". Then the yielding datum strain boundaries are calculated, from where on any parallel movement of the strain distribution would not bring any more changing of stresses. Naturally these yielding boundaries are only necessary, if for the steel elements an ideal plastic plateau without any strain hardening is given. Comparing the ultimate strain boundaries with the yielding ones leads to the decisive strain boundaries. The lower and upper ultimate normal force in the steel section depending on the given curvature к can then be calculated with the procedure „steelstrain" as shown in Fig.4-15.

| ultimate strain boundaries: | $\begin{aligned} \mathrm{e} 01 \mathrm{a} & =\text { datum strain where } \varepsilon_{\text {u,compr }} \\ \mathrm{e} 02 \mathrm{a} & =\text { datum first element reached } \\ & \mathrm{e} 01 \mathrm{a}<\mathrm{e} 02 \mathrm{a} \end{aligned}$ |
| :---: | :---: |
| yielding strain boundaries: | $\begin{aligned} \text { e01ya } & =\text { datum strain when } f_{\text {yd,compr }} \text { of last element reached } \\ \text { e02ya } & =\text { datum strain when } f_{\text {yd,tension }} \text { of last element reached } \\ & \text { e01ya <e02ya } \end{aligned}$ |
| decisive strain boundaries | $\begin{aligned} e 01 m a & =\max (e 01 a, e 01 y a) \\ \text { e02ma } & =\min (e 02 a, e 02 y a) \end{aligned}$ lower limit <br> upper limit <br> if e01ma>e02ma no solution can be found for the given curvature because the given ultimate strain limits cannot be met in any case |
| ultimate force limits: |  |

Fig.4-15 Strain boundaries from procedure „steellimit"

## Iteration methods



Fig.4-16 Iteration methods

In the second step the iteration for the corresponding datum strain in between the decisive strain boundaries will start, if the given normal force $N_{i a}$ lies in between the just calculated limits Nu1a and Nu2a. This is done within the procedure „itNsteel" in the following iterative way:

Starting with the boundaries e01ma and e02ma the actual normal force for the bisection point of these strains is calculated using the procedure "steelstrain". Then the strain boundaries are adapted so that the searched normal force lies in between the new boundaries. This iteration based on the bisection method is repeated until one of the following criteria is fulfilled: Either the actual normal force reaches the prescribed one with a given accuracy or the change of strains is smaller than a given tolerance or the iteration time exceeds a preset limit. A summary of the iteration accuracy criteria can be found at the end of chapter 4.2.

The method of halving the iteration interval generally has been chosen because of the big variation of gradients of the analysed curves, resulting from the relatively flat strain hardening plateau. Especially for such nearly bilinear functions the regula falsi or the Newton-method showed to be unstable, though for curves with a relatively constant gradient these methods would converge much faster (Fig.4-16).


Fig.4-17 Iteration example

Fig.4-17 shows the iteration and the results for the example section. It can clearly be seen that the relatively flat parts of the $N_{i a}-\varepsilon_{0 a}$-curve, which result from the strain hardening of the steel elements beyond reaching the design yield strength, dominate this function and therefore drastically slow down the iteration process. However thanks to the quick computers the iteration time anyway keeps really acceptable. Such an iteration plateau could be avoided by using material laws with a flat plateau (without strain hardening).

Apart from the possibility to make interpolations within the $N_{i a}-M_{a}$-curve the described procedure „Nsteeliteration" e.g. also can be used to iterate manually the ultimate limit of bending $\kappa_{u}$ (when $\varepsilon_{u}$ of the first element is reached) or to determine the connection between curvature ( $\kappa$ ) and neutral axis (N.A.) or bending moment ( $\mathrm{M}_{\mathrm{a}}{ }^{\prime}$ ) as illustrated in Fig.4-18 for the example section. Of course for the special case that $\mathrm{N}_{\mathrm{ia}}=0$ the relative bending moment $\mathrm{M}_{\mathrm{a}}$ (related to the datum line) is equal to the absolute one $\mathrm{M}_{\mathrm{a}}$.

$$
N_{i \mathrm{i} a}=0 \xrightarrow{\text { Nsteeliteration }} \begin{array}{ll}
\mathrm{M}_{\mathrm{el}}= \pm 145.0 \mathrm{kNm} \\
N . A \cdot{ }_{e l}=56.88 \mathrm{~mm} \\
\kappa_{e l}= \pm 10.403 \% / \mathrm{m}
\end{array} \quad \begin{aligned}
& M_{\mathrm{u}}= \pm 244.1 \mathrm{kNm} \\
& \mathrm{~N} \cdot \mathrm{~A}_{\mathrm{e}}=13.43 \mathrm{~mm} \\
& \kappa_{\mathrm{u}}= \pm 496.1 \% / \mathrm{m}
\end{aligned}
$$




Fig.4-18 Results using the procedure „Nsteeliteration"
4.2.3.4 N -M-relation for a defined curvature (NMstee)


Based on the just described strain iteration the full range between the force limits Nu1a and Nu2a can be analysed for a defined curvature $\kappa$ (procedure "NMsteel"). First the strain and force limits are calculated using the procedure "steellimit". Then the range between Nu1a and Nu2a is subdivided into 40 parts and for each normal force the corresponding datum strain $\varepsilon_{0 a}$ will be iterated with the procedure „itNsteel". Afterwards for the found solution the bending moment $M_{a}$ ' and the location of the neutral axis N.A.a are evaluated with the procedure "steelstrain". Finally the elastic and fully plastic steel section properties are calculated with the procedure "qswertesteel" based on the formulae shown in Fig.4-19.
reference modulus $E_{\text {ref }}=E_{\text {first teel element }}$
elastic neutral axis ENA $=\frac{\sum b_{i} \cdot h_{i} \cdot y_{s, i} \cdot \frac{E_{1}}{E_{\text {eq }}}}{\sum b_{i} \cdot h_{i} \cdot \frac{E_{i}}{E_{\text {ted }}}}$
plastic neutral axis PNA ... itNsteel $\left(\kappa= \pm \infty, N_{i a}=0\right)$
Area $A=\sum b_{i} \cdot h_{i}$
Shear area $A_{v c}=\sum_{\text {web elements }} b_{i} \cdot h_{i}$
Moment of inertia $I=\sum\left(\frac{b_{i} \cdot h_{i}^{3}}{12}+b_{i} \cdot h_{i} \cdot\left(\text { ENA }-y_{s_{i}}\right)^{2}\right) \cdot \frac{E_{i}}{E_{\text {ret }}}$
upper res.moment $W_{o}=\frac{I}{y_{\text {max }}-E N A}$
lower res.moment $\quad W_{u}=\frac{I}{E N A-y_{\text {min }}}$
first yielding $y_{\text {el }}$ when $\frac{\frac{E_{\text {ed }}}{E_{i}} \cdot f_{y, i}}{\mid y_{i}-\text { ENA } \mid} \rightarrow$ min
elastic curvature limit $\kappa_{e l}\left(N_{a}=0, V\right)=\min \left(\frac{f_{y v}}{\gamma_{a} \cdot E_{i} \cdot\left|y_{e l}-E N A\right|}\right)$
elastic limit of bending $M_{e l}\left(N_{a}=0, V\right)=E_{\text {ref }} \cdot I \cdot \kappa_{\text {el }}$
ideal plastic normal force $N_{p l}=\sum b_{i} \cdot h_{i} \cdot \frac{f_{y v i}}{\gamma_{a}}$
ideal plastic bending moment $M_{p l} \ldots$ itNsteel $\left(\mathrm{k}= \pm \infty, N_{\mathrm{ia}}=0\right)$
plastic vertical shear force $\mathrm{V}_{\mathrm{pl}}=\sum_{\text {web elements }} b_{i} \cdot h_{i} \cdot \frac{f_{y, i}}{\gamma_{a}} \cdot \frac{1}{\sqrt{3}}$
(Attention: buckling is not taken into consideration !)

Fig.4-19 Formulae for the calculation of the steel sectional properties (qswertesteel)

Two special buttons have been arranged to enable also a quasi-fully-plastic analysis. When clicking on „plastic sagging" or "plastic hogging" the curvature $\kappa$ will be set equal to $\pm 10^{5} \% / \mathrm{m}$ and all stress-strain diagrams are modified temporarily in that way, that any strain hardening is cut and an infinite yielding plateau without any limiting strain is provided.


Fig.4-20 Window for the presentation of steel results (eps.frm)

In the following the results from calculations for the example section using $\kappa=10 \% / \mathrm{m}$ and $\mathrm{V}=0 \mathrm{kN}$ are described. The progress of iteration will be indicated by a flood box. After the termination of the calculation the steel input window will be closed and the window for the presentation of the steel results is shown (Fig.4-20). It comprises the input data (table of elements, safety-factors, ...) an image of the steel section, the actual strain and stress distribution and a table and a graph of the calculated $\mathrm{N}_{\mathrm{ia}}-\mathrm{M}_{\mathrm{a}}{ }^{\prime}$ interaction. Those parts of the interaction curve drawn in green represent the elastic range, the non-elastic parts are drawn in black. The interaction table is combined with the possibility for a linear interpolation in view of the neutral axis N.A.a, $\mathrm{N}_{\mathrm{ia}}$ and $\mathrm{M}_{\mathrm{a}}$ '. Clicking on „sec.prop." switches to the window including the elastic and ideal plastic section properties calculated according to Fig.4-19.

To make the coherence between neutral axis, normal force and bending moment accessible to the user and therefore to get him the feeling where strengthening of the section would be economic or not special tools have been provided: Clicking on one line within the $N_{i a}-\mathrm{M}_{\mathrm{a}}$-table will produce the corresponding N.A. within the section image, the strain and stress distribution and a crosshair accentuating the actual location within the interaction curve. This graphical illustration will also be started after a linear interpolation or when following the interaction curve with the computer-mouse while the left mouse-button is pressed.

Using the button „Print results" will send the following four sheets to your printer: The first sheet (see page 174) contains a drawing of the actual steel section and the analysed range of strain distributions. Unfortunately when taking strain hardening into consideration the decisive strain limits become relatively large (depending on the given ultimate strain values $\varepsilon_{u}$ ) and therefore the strain diagram with decreasing $\kappa$ more and more deforms from a parallelogram to a rectangle loosing optical clearness. At the bottom of this first sheet the $\mathrm{N}_{\mathrm{ia}}-\mathrm{M}_{\mathrm{a}}$ ' diagram will be displayed. The grey line represents the elastic range, the black one the non-elastic area. At the second sheet the interaction table will be given (see page 175). The elastic or ideal-plastic cross-sectional properties are printed on sheet no. 3 (page 176) and finally on sheet no. 4 you will find the full input data of the actual section.

To make the results also accessible to other programs they can be saved as a sequential data file with the button "Save results". That will activate the file management window (Fig.4-4). The default file name is built up by that of the section (*.pro) and the extension "eps" (elasto-plastic steel). The output file of the example calculation can be found on page 177.




## Elasto-plastic results - steel section

Section:,"hat-shaped beam"
kapa [o/oo / m]=, 10
interacting vertical shear force $[\mathrm{N}]=, 0$
Savety factor steel:,1.1
Table of elements (No,x,y,w,h,fyk,E/1000,fu,eu,flange=0/web=1) [N,mm]

| 1 | 50 | 0 | 250 | 15 | 355 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 175 | 15 | 10 | 200 | 380 |
| 3 | 185 | 205 | 45 | 10 | 380 |
| 11 | 300 | 0 | 250 | 15 | 355 |
| 12 | 415 | 15 | 10 | 200 | 380 |
| 13 | 370 | 205 | 45 | 10 | 380 |

N-M' Interaction Table (y[mm],N[kN],M'[kNm],e0[0/oo])

| -9785.00 | -5456.930 | 291.197 | -97.850 | non-elastic |
| :---: | :---: | :---: | :---: | :---: |
| -7775.59 | -5177.551 | 281.142 | -77.756 | non-elastic |
| -5920.74 | -4919.665 | 271.860 | -59.207 | non-elastic |
| -3911.33 | -4640.286 | 261.805 | -39.113 | non-elastic |
| -1901.91 | -4360.908 | 251.750 | -19.019 | non-elastic |
| -146.09 | -4086.523 | 242.884 | -1.461 | non-elastic |
| -127.98 | -3819.850 | 240.816 | -1.280 | non-elastic |
| -112.28 | -3552.793 | 238.315 | -1.123 | non-elastic |
| -96.58 | -3275.417 | 235.276 | -0.966 | non-elastic |
| -80.88 | -2987.722 | 231.536 | -0.809 | non-elastic |
| -66.39 | -2712.999 | 227.322 | -0.664 | non-elastic |
| -53.11 | -2453.446 | 222.704 | -0.531 | non-elastic |
| -39.82 | -2186.505 | 217.263 | -0.398 | non-elastic |
| -26.54 | -1912.176 | 210.901 | -0.265 | non-elastic |
| -13.26 | -1630.458 | 203.520 | -0.133 | non-elastic |
| -0.58 | -1354.655 | 195.433 | -0.006 | non-elastic |
| 11.50 | -1085.726 | 186.707 | 0.115 | non-elastic |
| 23.57 | -810.692 | 176.906 | 0.236 | non-elastic |
| 35.05 | -543.753 | 166.533 | 0.351 | non-elastic |
| 47.12 | -252.680 | 153.568 | 0.471 | non-elastic |
| 56.87 | -0.004 | 139.403 | 0.569 | elastic |
| 67.65 | 280.579 | 123.445 | 0.677 | elastic |
| 78.52 | 563.587 | 107.349 | 0.785 | elastic |
| 88.78 | 830.873 | 92.147 | 0.888 | elastic |
| 99.05 | 1098.159 | 76.945 | 0.991 | elastic |
| 109.92 | 1381.168 | 60.849 | 1.099 | elastic |
| 120.18 | 1648.454 | 45.647 | 1.202 | elastic |
| 130.44 | 1915.739 | 30.445 | 1.304 | elastic |
| 141.31 | 2198.748 | 14.349 | 1.413 | elastic |
| 151.58 | 2466.034 | -0.853 | 1.516 | elastic |
| 164.26 | 2737.881 | -19.426 | 1.643 | non-elastic |
| 189.62 | 3009.609 | -54.107 | 1.896 | non-elastic |
| 218.60 | 3281.087 | -92.772 | 2.186 | non-elastic |
| 252.41 | 3553.361 | -135.406 | 2.524 | non-elastic |
| 295.88 | 3833.088 | -183.810 | 2.959 | non-elastic |
| 363.51 | 4110.935 | -237.175 | 3.635 | non-elastic |
| 2116.91 | 4380.792 | -251.439 | 21.169 | non-elastic |
| 4126.33 | 4660.170 | -261.494 | 41.263 | non-elastic |
| 6135.74 | 4939.548 | -271.549 | 61.357 | non-elastic |
| 7990.59 | 5197.436 | -280.831 | 79.906 | non-elastic |
| 10000.00 | 5476.814 | -290.886 | 100.000 | non-elastic |
| 20 |  |  |  |  |

## Section properties:

reference E-modulus [kN/mm²]:,210
E.N.A.from bottom line [mm]:,56.875
first yielding at [mm]:,215
Area [mm2]:,12400
Shear area Avc [mm2]:,4000
Inertia [cm4]:,6638.224
upper Resis.Moment [cm3]:,419.8086
lower Resis.Moment [cm3]:,1167.16
El.limit of bending [kNm]:,145.0248
El.limit of kapa [o/oo/m]:,10.4033
Plastic normal force [kN]:,4113.182," without buckling-check"
Plastic shear force (web-parts): [kN],797.7931," without buckling-check"

Applying the formulae given in Fig.4-19 this procedure (4.2.3.4) can also be used e.g. to determine the $\mathrm{M}_{\mathrm{Rd}}-\mathrm{V}_{\mathrm{Sd}}$ or the $\mathrm{N}_{\mathrm{Rd}}-\mathrm{V}_{\mathrm{Sd}}$ interaction of a steel section. In Fig.4-21 the maximum plastic sagging moment of the example section has been evaluated for different values of interacting vertical shear. Fig.4-22 shows the interaction between normal force resistance and actual vertical shear. As described in Fig.4-9 the interaction between normal stresses and vertical shear stresses leads to a fictitious reduction of the stress-strain diagram of the steel web elements.


Fig.4-21 Interaction between bending resistance and vertical shear


Fig.4-22 Interaction between normal force resistance and vertical shear

### 4.2.4 Concrete section

### 4.2.4.1 Section input

The concrete section window, shown in Fig.4-23 and Fig.4-26, first of all enables the input of the reinforced concrete section part consisting of up to 10 rectangular concrete elements and 10 layers of reinforcement. It will be assumed that the overall concrete section is shear-connected to the steel part.


Fig.4-23 Window for the reinforced concrete section part - concrete elements (prc.frm)

First the input window for the 10 concrete elements (button "CONCRETE") will be opened as illustrated in Fig.4-23. In view of the possibility to mirror the section horizontally the final concrete section may consist of up to 20 concrete elements. The 10 basic elements are marked with different colours. To point out the mirrored elements they are drawn with a more pale colour, what is also stressed in the "Mirror"-box. For each element the coordinates of one corner ( $\mathrm{x}, \mathrm{y}$ ) and the width and height ( $\mathrm{b}, \mathrm{h}$ ) to the other corner can be defined. Also negative values are accepted, however then the $x$ and y-coordinates will automatically be re-calculated for the lower left corner. To facilitate the input the coordinates of the actual mouse-position are indicated at the top of the drawing. A single click within the drawing will write the coordinates of the nearest existing element corner (also steel elements are accepted) to the $x$ - and y-boxes. A
double-click is understood as the second corner of the actual element. Therefore the actual element width and height then are calculated as the distance between the given $x$ - and $y$-values and the nearest element corner from the double-click position. Attention has to be paid that the corners of mirrored elements cannot be caught; they will automatically be transformed to the basic element corner.

Apart from the geometrical properties also the material properties can be defined for each concrete element separately. The stress-strain diagram is based on the following input data: Characteristic compressive strength $f_{c k}$ and mean tensile strength $f_{c t m}$. Further it depends on the global partial safety factor of concrete $\gamma_{\text {concr }}=\gamma_{c}$ (default=1.5) and the stress factor $\alpha_{c}$ (default=0.85). The definition of the bi-linear stress-strain diagram for concrete elements used in this program is summarised in Fig.4-24 based on [182]. It can be seen that for the bare concrete no tensile strength is taken into consideration. The effective tensile strength $f_{c t, \text { eff }}$, which can be gained from the mean value $f_{\text {ctm }}$ multiplied by a factor, is only considered for the tension stiffening effect of the reinforcement embedded in this concrete element. The strains of cracking, elastic limit and crushing have generally been defined to $0.15,1.35$ and $3.5 \%$ independently of the concrete grade. The corresponding elastic modulus can be calculated according to the following formula and will be displayed for each element. So to make an elastic calculation the compressive strength $f_{c k}$ of these elements has to be chosen so that a given value of the elastic modulus $\mathrm{E}_{\mathrm{c}}$ results from the following relationship.

$$
E_{c}=\frac{\alpha_{c} \cdot f_{c k}}{\gamma_{c} \cdot \frac{1.35}{1000}}
$$



Fig.4-24 Stress-strain diagram of concrete elements

The actual stress-strain diagram of each concrete element can be seen true to scale when clicking on the „ $\sigma-\varepsilon$ "-button (Fig.4-25). There will be the possibility to calculate the corresponding stress to a given strain and vice versa by a linear interpolation.

According to $[59,109]$ the contribution of concrete to the overall vertical shear resistance has generally been neglected because it is much smaller than the resistance of the steel section (web elements). This assumption has been made especially in view of conventional composite beams with relatively thin slabs on top of deep steel beams. However also in view of slim floors in the joints (being the decisive locations for shear design) the whole vertical shear force anyway has to be carried by the bare steel connection. Therefore at these decisive points all the vertical shear force yet has to be assigned to the steel webs alone. So this justifies the assumption to ignore the concrete contribution to vertical shear.

The longterm redistribution of forces from the concrete to the reinforcing steel due to creep and shrinkage generally has not been taken into consideration because of its minor influence.


Fig.4-25 Window for the actual stress-strain diagrams of concrete elements (stoffgc.frm)


Fig.4-26 Window for the reinforced concrete section part - reinforcement layers (prc.frm)

Clicking on the button „REINFORCEM." switches from the concrete input window to that for the reinforcement (Fig.4-26). The 10 layers are marked with different colours. For each layer the y-coordinate has to be defined. To facilitate the input the coordinates of the actual mouse-position are indicated at the top of the drawing. A single click within the drawing will write the $y$-coordinate of the nearest existing element corner (steel and concrete elements) to the y-box. Further the diameter and number of reinforcing bars at this level have to be given. In the drawing the rebars are shown true to scale however simplifying with an equidistant spacing in horizontal direction (the x-values anyway do not have any meaning for the calculations). Attention has to be paid that the reinforcement in the calculations is regarded to be concentrated at the given y-value. The diameter only is necessary to evaluate the area of reinforcement, the strain distribution within the rebars and therefore their bending stiffness will be neglected.

Apart from the geometrical properties also the material properties can be defined for each reinforcement layer separately. The stress-strain diagram of a bare (unembedded) reinforcing bar is assumed to be symmetrical for tension and compression (Fig.4-27) and is based on the following input data: Young modulus $\mathrm{E}_{\mathrm{s}}$, characteristic yielding strength $f_{s k}$, ultimate strength $f_{u}$ and ultimate strain $\varepsilon_{u}$. Further it depends on the global partial safety factor of reinforcement $\gamma_{\text {reinf. }}=\gamma_{r}($ default=1.15).


Fig.4-27 Stress-strain diagram of bare unembedded reinforcement

As already mentioned the concrete itself will not be taken into consideration in tension but it significantly contributes to the behaviour of the reinforcement. This strengthening effect of the reinforcement in tension by the concrete between the cracks is called "tension stiffening effect" [72,73,177]. The mechanical background can easily be explained for a reinforced concrete member in pure tension. Until the tensile strength of the weakest concrete cross section is reached the strains of concrete and reinforcement are equal and there are no bond stresses in between. At the state of the first crack in the cracked cross section the full crack force $N=N_{1}$ has to be carried by the bare reinforcement leading to an increase of strain in comparison to the composite section between the cracks. At the cracked section the concrete strain is zero. Within a transmission length $\ell_{t}$ to both sides of the crack due to bond stresses the strain of the reinforcement gradually is reduced and that of the concrete gradually is raised until at the ends of this force transmission the strains of reinforcement and concrete again are equal and the total force $N=N_{1}$ is shared between these two section parts. Due to the crack the elongation of the member increases in comparison to the uncracked state and can be calculated by integrating the strain distribution over the member length. Increasing the tension force beyond $N_{1}$ will lead to the next crack in the second weakest cross section with the same effect of bond stresses and strain distribution within the transmission length. This quick formation of cracks can be continued until the distance between the cracks approximately is equal to the transmission length. From this state on the bond stresses along the half crack distance are too small to activate the tensile strength in the concrete and therefore to initiate new cracks. This occurs at the force limit $\mathrm{N}_{2}$. Within the following range of stabilised cracks there are nearly no new cracks and the whole increase of force is taken over by the bare reinforcement. The average transmission length is assumed to be $2 / 3$ of $\ell_{\mathrm{t}}$, so the average crack distance is 2 times $2 / 3 \ell_{t}$. The average steel strain $\varepsilon_{s m}$ taking into consideration the tension stiffening effect can be got by integrating the strain distribution. It obviously will be smaller than the bare unembedded steel strain. The third bend $\left(\mathrm{N}_{3}\right)$ of the embedded force-strain diagram is linked to the yield stress of the reinforcement. Finally it has to be emphasised that the ultimate strain of an embedded rebar is drastically reduced in comparison to that of bare steel. This negative effect gains importance with decreasing degree of
reinforcement. The mechanical principles of tension stiffening are summarised in Fig.4-28 and Fig.4-29.


Fig.4-28 Tension stiffening effect in a reinforced concrete tension member

$$
\begin{aligned}
& \ell_{\mathrm{t}} \Rightarrow \quad \varepsilon_{\mathrm{sm}}=\varepsilon_{\mathrm{s} 2}-\mathrm{B} \cdot \Delta \varepsilon_{\mathrm{sr}} \\
& \ell_{\mathrm{tm}}=\frac{2}{3} \cdot \ell_{\mathrm{t}} \Rightarrow \varepsilon_{\mathrm{sm}}=\varepsilon_{\mathrm{s} 2}-\frac{2}{3} \cdot \mathrm{~B} \cdot \Delta \varepsilon_{\mathrm{sr}}=\varepsilon_{\mathrm{s} 2}-\beta \cdot \Delta \varepsilon_{\mathrm{sr}} \quad \beta_{\text {short term }}=0.4 \\
& \varepsilon_{\mathrm{sm}} \ldots \text { average steel strain considering tension stiffening } \\
& \varepsilon_{\mathrm{s} 2} \quad . . . \text { steel strain in the crack (state II) } \\
& \Delta \varepsilon_{\mathrm{sr}}=\varepsilon_{\mathrm{sr} 2}-\varepsilon_{\mathrm{sr} 1}=\frac{\mathbf{f}_{\mathrm{ct}, \mathrm{eff}} \cdot \mathbf{A}_{\mathrm{ct}}}{\mathbf{E}_{\mathrm{s},} \cdot \mathbf{A}_{\mathrm{s}}} \\
& \varepsilon_{\text {sr1 }}=\frac{f_{c t, \text { eff }}}{\frac{E_{s}}{n}} \quad n=\frac{E_{s}}{E_{c, \text { tension }}} \\
& \varepsilon_{\mathrm{sr} 2}=\frac{N_{1}}{E_{s} \cdot A_{s}} \quad N_{1}=f_{c t, \text { eft }} \cdot\left(A_{c t}+n \cdot A_{s}\right)
\end{aligned}
$$

Fig.4-29 Mechanical background of tension stiffening

Following the rules given in [62]-Annex $L$ the mathematical model shown in Fig.4-30 and Fig.4-31 has been adopted for the described computer program. Only the strain of the first crack as a minor modification generally has been defined with $0.15 \%$. As a simplification the relatively small steel area will not be subtracted from the concrete ( n instead of ( $n-1$ )).

$$
\begin{aligned}
& \varepsilon_{1}=\frac{0.15}{1000} \\
& N_{3}=A_{s} \cdot \frac{f_{s k}}{\gamma_{r}} \\
& N_{1}=E_{s} \cdot A_{s} \cdot \varepsilon_{1}+A_{c t} \cdot f_{c t, \text { eff }} \leq N_{3} \\
& N_{0} \leq N_{1} \quad \text { normal force due to shrinkage decreases between } \varepsilon_{1} \text { and } \varepsilon_{2} \\
& N_{2}=1.3 \cdot N_{1} \leq N_{3} \\
& \Delta \varepsilon=\frac{f_{c t, \text { eff }} \cdot A_{c t}}{E_{s} \cdot A_{s}} \quad \Delta N=E_{s} \cdot A_{s} \cdot \beta \cdot \Delta \varepsilon=\beta \cdot f_{c t, \text { eff }} \cdot A_{\mathrm{ct}} \quad \beta_{\text {short term loading }}=0.4 \\
& \varepsilon_{2}=\frac{N_{2}-\Delta N}{E_{s} \cdot A_{s}} \\
& \varepsilon_{3}=\frac{N_{3}-\Delta N}{E_{s} \cdot A_{s}}=\varepsilon_{y}-\beta \cdot \Delta \varepsilon \\
& N_{4}=A_{s} \cdot \frac{f_{u}}{\gamma_{r}} \\
& \varepsilon_{4}=\varepsilon_{3}+\delta \cdot\left(1-\frac{N_{1}}{N_{3}}\right) \cdot\left(\varepsilon_{u}-\varepsilon_{y}\right)
\end{aligned}
$$

$$
\delta=0.8 \quad \text { for ductile steel with } \varepsilon_{u}>5 \% \text { and } \frac{F_{u}}{f_{s k}}>1.08
$$

Fig.4-30 Modelling of the tension stiffening effect - formulae

When setting the area of concrete in tension $\left(\mathrm{A}_{\mathrm{ct}}\right)$ to zero the formulae given in Fig.4-30 lead directly to the basic stress-strain diagram of the bare reinforcement. Only the ultimate strain $\varepsilon_{4}$ would be smaller than $\varepsilon_{u}$. This inconsistency has been corrected in the computer program: if $A_{c t}>0 \varepsilon_{4}=\varepsilon_{u}$ otherwise the formula given in Fig.4-30 is used. Attention has been paid to the following special cases: If $N_{2}$ exceeds the yielding force $\left(N_{3}\right)$ in case of a low degree of reinforcement it will be set equal to $N_{3}$. If even the uncracked force in tension $\left(\mathrm{N}_{1}\right)$ exceeds the yielding force of the bare reinforcement $\left(\mathrm{N}_{3}\right)$ that indicates that the area of reinforcement has been chosen below the minimum reinforcement. This fact will be indicated by a warning. However to enable still a further hypothetical calculation $N_{1}$ in this case will be set equal to $N_{3}$ and $\varepsilon_{4}$ will be taken equal to $\varepsilon_{u}$.


Fig.4-31 Modelling of the tension stiffening effect - graph

Both in the drawings and the tables the resulting normal force explicitly will be set together by the shares of the bare steel, the concrete in tension between the cracks and occasionally also of the shrinkage influence. For the fictitious normal force due to shrinkage an input box has been arranged within the reinforcement window („ $\mathrm{N}_{\text {shrink. }}$.). Although they influence the response of the reinforcement in tension the $\beta$-value as well as the ratio between effective and mean tensile strength of concrete ( $f_{c t, e f f} / f_{c t m}$ ) have to be set on the concrete input window, as they are primarily linked to the concrete in tension between the cracks.

Strictly the described procedure only is valid for reinforced concrete members in pure tension. So as a simplification for the computer program the concrete section part in tension and bending has been divided into substituting tension members consisting of the reinforcement layers with surrounding concrete. The resulting tension forces will be taken from the strain-force diagram of the embedded steel for the strain at the reinforcement centre. Concrete in tension is taken into consideration by a modification of the reinforcement behaviour; there will not be calculated a further separate tension
stress-block of the concrete itself. The remaining problem was to define the effective area of concrete contributing in tension ( $\mathrm{A}_{\mathrm{ct}}$ ) to the reinforcement bars (see Fig.4-32). For that purpose first a square of $15 \mathrm{~d}_{\mathrm{s}}$ is drawn around each rebar as the maximum contributing concrete area. This value has been taken from [30] where an area of 2 times $7.5 \mathrm{~d}_{\mathrm{s}}$ is specified to influence the crack initiation at the surface. Strictly until the first crack occurs for a bare tension member the whole area of concrete in tension would have to be considered. Due to the slender concrete elements in composite members the relatively high upper limit of $7.5 \mathrm{~d}_{\mathrm{s}}$ anyway very rarely becomes decisive and therefore has been accepted as a compromise. This maximum area of contributing concrete around each rebar has to be reduced if the section borders are reached, if contributing areas of several rebars are overlapping (in that case the distance in between is shared corresponding to the ratio of reinforcement areas) or if it would reach into the compression area. This is the case if the neutral axis lies in between the $7.5 \mathrm{~d}_{\mathrm{s}}$ radius of a reinforcement layer. Finally in the special case that several reinforcement layers have the same $y$-value only for that layer with the maximum steel area the tension stiffening effect will be taken into consideration. The other layers at this level then will be calculated as bare steel layers.

Fig.4-32 Definition of the contributing area of concrete in tension

The actual strain-force diagram of each reinforcement layer can be seen true to scale when clicking on the "N- $\varepsilon$-"button (Fig.4-33). There the actual layer is painted in black, the contributing concrete area in yellow. The boxes for $\varepsilon$ and N enable a linear interpolation between strains and corresponding forces. Fig.4-33 shows the $\mathrm{N}-\varepsilon$ diagram of the example section using $\gamma_{r}=1.15, \beta=0.4$ and $f_{c t, e f f} / f_{c t m}=1$. Fig. $4-34$ gives the detail without the yielding or strain-hardening part (click on diagram). To take into consideration an eventual reduction of the tension stiffening concrete area (indicated in yellow) if the actual neutral axis reaches into this yellow area in Fig.4-33 a neutral axis input box has been provided.


Fig.4-33 Window for the actual strain-force diagrams of rebars (stoffgr.frm)


Fig.4-34 Window for the actual strain-force diagrams of rebars (stoffgr.frm)

Before returning from the concrete section window to the section management window or before starting any calculation it will be checked automatically if elements are overlapping and therefore these areas would be calculated more than once. Further after each element input the entire composite cross section automatically will be moved so that the minimum $x$-value of the entire section is equal to zero.

Based on the so-defined concrete section including geometry and material properties the following three calculations or iterations can be carried out:

- internal forces to a given strain distribution
- strain distribution to a given normal force
- N-M-relation for a defined curvature
4.2.4.2 Internal forces to a given strain distribution (concstrain)


The actual linear strain distribution within the reinforced concrete section can be defined by the strain at height of the datum line ( $\mathrm{y}=0=$ steel bottom) $\varepsilon_{0 c}$ and the curvature $\kappa$ as shown in Fig.4-35.


Fig.4-35 Linear strain distribution
Based on that the global procedure ,,concstrain" calculates the corresponding stresses of each concrete element and the corresponding forces of each reinforcement layer based on the material properties. The following indicator flags are set (Fig.4-36): elastic („el") means that for a reinforced concrete section all concrete strains are in between -1.35 and $0.15 \%$. For a plain concrete section even no tension $(\varepsilon=0)$ will be allowed. Further the yield strain $\varepsilon_{y}$ of all rebars in compression must not be exceeded. The non-elasitic flag (,ne") is set if the strain in at least one concrete point exceeds $-1.35 \%$ in compression or passes $0.15 \%$ in tension. Non-elastic will also be indicated if the strain in at least one rebar exceeds $\varepsilon_{y}$. If the concrete strain in compression reaches $-3.5 \%$ crushing of concrete („cr") will be indicated and for the case that the provided reinforcement is less than the minimum reinforcement „beyond ultimate strain" („**") is shown for $\varepsilon>\varepsilon_{3}$.

## concrete

$$
\begin{array}{lr}
\text { non elastic } & 0.15 \% \circ(0 \%)<\varepsilon \\
\text { elastic } & -1.35 \% \circ<\varepsilon<0.15 \% \circ(0 \% \circ) \\
\text { non - elastic } & -3.5 \% \circ<\varepsilon<-1.35 \% \circ \\
\text { crushing of concrete } & \varepsilon<-3.5 \%
\end{array}
$$

## reinforcement

non - elastic
elastic
non-elastic
$">\varepsilon_{u}$ " if $A_{s}<A_{s, \min }$


$$
-\varepsilon_{y}<\varepsilon<0.15 \%
$$

$0.15 \%<\varepsilon$
$\varepsilon_{3}<\varepsilon$
Fig.4-36 Indicator flags
An integration of the stresses and forces over the whole concrete section leads to the normal force $\mathrm{N}_{\mathrm{ic}}$ (compression positive). Integrating the stresses and forces multiplied by their lever arms to the steel bottom results in the moment $M_{c}$ ', which of course is related to this datum line $\mathrm{y}=0$ as described in [106] (sagging moments positive). Finally also the $y$-value of the neutral axis (N.A.c) is declared.

Fig.4-37 illustrates this basic calculation for the example section leading from strains to stresses and internal forces.


CONCRETE elements:

| el.no. | $\varepsilon_{\text {bottom }}\left[\%{ }^{\circ}\right]$ | $\varepsilon_{\text {top }}\left[\%{ }^{\circ}\right]$ | $\sigma_{\text {bottom }}\left[\mathrm{N} / \mathrm{mm}^{2}\right]$ | $\sigma_{\text {top }}\left[\mathrm{N} / \mathrm{mm}^{2}\right]$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.535 | -0.265 | $\mathbf{0 . 0 0}$ | $\mathbf{- 3 . 3 4}$ | non-elastic |
| 11 | 0.535 | -0.265 | 0.00 | -3.34 | non-elastic |

REINFORCEMENT layers:

| el.no. | $\varepsilon[\%]]$ | $F_{\text {steel }}[\mathrm{kN}]$ | $\mathrm{F}_{\text {concr }}[\mathrm{kN}]$ | $\mathrm{F}[\mathrm{kN}]$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $\mathbf{0 . 0 3 5}$ | 4.36 | 22.24 | $\mathbf{2 6 . 6 0}$ | elastic |

Fig.4-37 Calculation example

### 4.2.4.3 Strain distribution to a given normal force (Nconcriteration)



The global procedure „Nconcriteration" goes the opposite direction iterating the corresponding strain distribution to a given normal force $\mathrm{N}_{\mathrm{ic}}$ and a given curvature $\kappa$. This problem is solved in two steps:

First the ultimate boundaries of datum strains (at $y=0$ ) depending on the curvature $\kappa$ and the ultimate strain values $\varepsilon_{u}$ of the concrete and reinforcement elements are derived within the procedure „conclimit". Then the yielding datum strain boundaries are calculated. These are the boundaries from where on any parallel movement of the strain distribution would not bring any changes of stresses or forces. Comparing the ultimate strain boundaries with the yielding ones leads to the decisive strain boundaries. The lower and upper ultimate normal force in the reinforced concrete section (depending on the given curvature $\kappa$ ) then can be calculated with the procedure "concstrain" as shown in Fig.4-38.


Fig.4-38 Strain boundaries from procedure „conclimit"

In the second step the iteration for the corresponding datum strain in between the decisive strain boundaries will start, if the given normal force $\mathrm{N}_{\mathrm{ic}}$ lies in between the just calculated limits Nu1c and Nu2c. This is done within the procedure „itNconcr" in the following iterative way:

Starting with the boundaries e 01 mc and e 02 mc the actual normal force for the bisection point of these strains is calculated using the procedure "concstrain". Then the strain boundaries are adapted so that the prescribed normal force lies within these new boundaries. This iteration based on the bisection method is repeated until one of the following criteria is fulfilled: Either the actual normal force attains the prescribed one with a given accuracy or the changes of strains lie below a given tolerance. A stop is also caused if the iteration time exceeds a stop-limit. That would indicate a jump of the normal force despite of a minor change of the datum strain due to the force within the reinforcement in combination with a big curvature. In that case the final solution of $\varepsilon_{0 c}$ is found by an interpolation. A summary of the iteration accuracy criteria can be found at the end of chapter 4.2.

Fig.4-39 shows the iteration and the results for the example section. It can clearly be seen that the relatively flat parts of the $\mathrm{N}_{\mathrm{ic}}-\varepsilon_{0 \mathrm{c}}$-curve, which result from the strain hardening of the reinforcement elements beyond the design yield strength, dominate this function and therefore drastically slow down the iteration process. However thanks to the quick computers the iteration time anyway keeps really acceptable. Such an iteration plateau could be avoided by using material laws with a flat plateau (without strain hardening).


Fig.4-39 Iteration example
Apart from the possibility to make interpolations within the $\mathrm{N}_{\mathrm{ic}}-\mathrm{M}_{\mathrm{c}}$ '-curve the described procedure „Nconcriteration" e.g. also can be used to iterate manually the ultimate limit of bending $\kappa_{u}$ (iteration until $\varepsilon_{u}$ of the first element is reached) or to determine the coherence between curvature ( $\kappa$ ) and neutral axis (N.A.) or bending moment ( $\mathrm{M}_{\mathrm{c}}{ }^{\prime}$ ) as illustrated in Fig.4-40 for the example section. Of course for the special case that $N_{i c}=0$ the relative bending moment $M_{c}$ ' (related to the datum line) is equal to the absolute one $\mathrm{M}_{\mathrm{c}}$.

$$
\mathrm{N}_{\mathrm{ic}}=0 \xrightarrow{\text { Nconcriteration }} \begin{array}{cc}
\mathrm{M}_{\mathrm{u}}=4.590 \mathrm{kNm} & \mathrm{M}_{\mathrm{u}}=-11.31 \mathrm{kNm} \\
\mathrm{~N} . \mathrm{A}_{\cdot \mathrm{u}}=265 \mathrm{~mm} & {\mathrm{~N} . A_{\cdot u}}=248.21 \mathrm{~mm} \\
\kappa_{\mathrm{u}}=+\infty & \kappa_{\mathrm{u}}=-\infty
\end{array}
$$

assuming a yield-plateau beyond $\varepsilon_{u}$ of reinf.






Fig.4-40 Results using the procedure „Nconcriteration" for further iterations

### 4.2.4.4 N -M-relation for a defined curvature

## (NMconcr)



Based on the just described strain iteration the full range between the force limits Nu1c and Nu2c can be analysed for a defined curvature $\kappa$ (procedure „NMconcr"). First the strain and force limits are calculated using the procedure "conclimit". Then the range between Nu2c and Nu1c is subdivided into 40 parts and for each normal force the corresponding datum strain $\varepsilon_{0 c}$ will be iterated with the procedure "itNconcr". Afterwards for the found solution the bending moment $\mathrm{M}_{\mathrm{c}}$ ' and the location of the neutral axis N.A.c are evaluated with the procedure „concstrain". Finally the fully plastic concrete section properties are calculated with the procedure "qswerteconcr" based on the formulae shown in Fig.4-41.


Fig.4-41 Formulae for the calculation of the concrete sectional properties (qswerteconcr)

Two special buttons have been arranged to enable also a quasi-fully-plastic analysis. When clicking on „plastic sagging" or „plastic hogging" the curvature $\kappa$ will be set equal to $\pm 10^{5} \% / \mathrm{m}$ and all stress-strain diagrams of the reinforcement layers are modified temporarily in that way, that any strain hardening is cut and an infinite yielding plateau without any limiting strain is provided.

In the following the results from calculations for the example section using $\kappa=10 \% / \mathrm{m}$ are described. The progress of iteration will be indicated by a flood box. After the termination of the calculation the concrete input window will be closed and the window for the presentation of the concrete results is shown (Fig.4-42). It comprises the input data (table of concrete elements and reinforcement layers, safety-factors, ...) an image of the concrete section, the actual strain and stress distribution and a table and a graph of the calculated $\mathrm{N}_{\mathrm{ic}}-\mathrm{M}_{\mathrm{c}}$ '-interaction. Those parts of the interaction curve drawn in green represent the elastic range, the non-elastic parts are drawn in black, the range of crushing concrete in compression is indicated with a red colour. Where even $\varepsilon_{u}$ is exceeded a white colour is used. The resulting force of each reinforcement layer also in the stress-drawing clearly is set together by the bare steel part (green), the contribution of concrete between the cracks (orange) and the shrinkage influence (brown). The interaction table is combined with the possibility for a linear interpolation in view of the neutral axis N.A.c, $\mathrm{N}_{\mathrm{ic}}$ and $\mathrm{M}_{\mathrm{c}}$. Clicking on „sec.prop." switches to the window of the ideal plastic section properties calculated according to the formulae shown in Fig.4-41.

To make the coherence between neutral axis, normal force and bending moment accessible to the user and therefore to get him the feeling where strengthening of the section would be economic or not special tools have been provided: Clicking on one line within the $N_{i c}-M_{c}$ '-table will produce the corresponding N.A. within the section image, the strain and stress distribution and a crosshair accentuating the actual location within the interaction curve. This graphical illustration will also be started after a linear interpolation or when following the interaction curve with the computer-mouse while the left mouse-button is pressed.


Fig.4-42 Window for the presentation of concrete results (epc.frm)

Using the button „Print results" will send the following four sheets to your printer: The first sheet (see page 197) contains a drawing of the actual concrete section and the analysed range of strain distributions. Unfortunately when taking strain hardening into consideration the decisive strain limits become relatively large (depending on the given ultimate strain values $\varepsilon_{\mathrm{u}}$ ) and therefore the strain diagram with decreasing $\kappa$ more and more deforms from a parallelogram to a rectangle loosing optical clearness. At the bottom of this first sheet the $\mathrm{N}_{\mathrm{ic}}-\mathrm{M}_{\mathrm{c}}$ ' diagram will be displayed. The thick grey line represents the elastic range, the thick black one the non-elastic area, the range of crushing concrete is drawn with a thin grey line and those areas beyond $\varepsilon_{u}$ are indicated with a very thin grey line. At the second sheet the interaction table will be given (see page 198). Only if $\kappa$ is infinite the ideal-plastic cross-sectional properties are printed on sheet no. 3 (page 199) and finally on sheet no. 4 you will find the full input data of the actual section.

To make the results also accessible to other programs they can be saved as a sequential data file with the button „Save results". The default file name is built up by that of the section (*.pro) and the extension „epc" (elasto-plastic concrete). The output file of the example calculation can be found on page 200.


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## Elasto-plastic results - shear-connected concrete section

Section:,"hat-shaped beam"
kapa [o/oo / m]=,10
Savety factor concrete:, 1.5
Stress factor concrete:, 0.85
Savety factor reinf.:, 1.15
Tension stiffening effect : fcteff/fctm =, 1
Tension stiffening effect: beta (short) $=, 0.4$
Table of concrete elements ( $\mathrm{No}, \mathrm{x}, \mathrm{y}, \mathrm{w}, \mathrm{h}, \mathrm{fck}, \mathrm{fctm}$ ) [ $\mathrm{N}, \mathrm{mm}$ ]

| 1 | 0 | 215 | 300 | 80 | 30 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 11 | 300 | 215 | 300 | 80 | 30 | 3 |

$\begin{array}{ccccccc}\text { Table of reinforcem. elements (No,y,no,d,fy, } \mathrm{E} / \mathbf{1 0 0 0} \text {, fu,eu, } \mathrm{N}(\text { shrink.) }) \\ 1 & 265 & 8 & 10 & 550 & 200 & 620\end{array}$
N-M' Interaction Table (y[mm],N[kN],M'[kNm],e0[o/oo])

| 4035.24 | -338.746 | -89.768 | 40.352 | non-elastic |
| :---: | :---: | :---: | :---: | :---: |
| 592.68 | -301.937 | -80.013 | 5.927 | non-elastic |
| 429.63 | -264.476 | -70.086 | 4.296 | non-elastic |
| 399.37 | -226.454 | -60.010 | 3.994 | non-elastic |
| 337.17 | -188.802 | -50.032 | 3.372 | non-elastic |
| 288.43 | -153.242 | -40.564 | 2.884 | non-elastic |
| 278.97 | -115.112 | -30.265 | 2.790 | non-elastic |
| 275.82 | -78.675 | -20.521 | 2.758 | non-elastic |
| 272.25 | -39.353 | -9.990 | 2.723 | non-elastic |
| 268.47 | 0.000 | 0.563 | 2.685 | non-elastic |
| 264.89 | 34.372 | 9.792 | 2.649 | non-elastic |
| 255.23 | 72.032 | 20.089 | 2.552 | non-elastic |
| 247.24 | 108.467 | 29.957 | 2.472 | non-elastic |
| 239.68 | 147.427 | 40.405 | 2.397 | non-elastic |
| 233.38 | 183.196 | 49.904 | 2.334 | non-elastic |
| 227.07 | 221.967 | 60.104 | 2.271 | elastic |
| 221.61 | 257.997 | 69.496 | 2.216 | elastic |
| 216.15 | 296.282 | 79.388 | 2.162 | elastic |
| 211.10 | 333.050 | 88.829 | 2.111 | elastic |
| 206.06 | 369.868 | 98.281 | 2.061 | elastic |
| 201.02 | 406.686 | 107.733 | 2.010 | elastic |
| 195.56 | 446.572 | 117.972 | 1.956 | elastic |
| 190.51 | 483.390 | 127.424 | 1.905 | elastic |
| 185.47 | 520.208 | 136.876 | 1.855 | elastic |
| 180.43 | 557.026 | 146.328 | 1.804 | elastic |
| 175.38 | 593.844 | 155.780 | 1.754 | elastic |
| 170.34 | 630.662 | 165.232 | 1.703 | elastic |


| 164.88 | 670.548 | 175.472 | $1.649 \quad$ elastic |
| :--- | :--- | :--- | :--- | :--- |


| 159.84 | 707.365 | 184.923 | 1.598 | non-elastic |
| :---: | :---: | :---: | :---: | :---: |
| 154.79 | 743.160 | 194.075 | 1.548 | non-elastic |
| 148.91 | 782.492 | 204.049 | 1.489 | non-elastic |
| 143.03 | 819.209 | 213.281 | 1.430 | non-elastic |
| 136.72 | 855.647 | 222.366 | 1.367 | non-elastic |
| 129.58 | 893.314 | 231.677 | 1.296 | non-elastic |
| 121.59 | 930.848 | 240.877 | 1.216 | non-elastic |
| 112.77 | 966.730 | 249.609 | 1.128 | non-elastic |
| 101.00 | 1005.419 | 258.993 | 1.010 | non-elastic |
| 83.35 | 1043.839 | 268.478 | 0.834 | non-elastic |
| 54.78 | 1080.173 | 278.086 | 0.548 | non-elastic |
| -267.96 | 1117.651 | 288.018 | -2.680 | crush.of concr. |
| -9735.00 | 1154.746 | 297.848 | -97.350 | crush.of concr. |

## Section properties:

Area concrete [cm2]:, 480
Area reinforcem. [mm2]:, 628.3185
end

### 4.2.5 Composite section

### 4.2.5.1 Section input

The composite cross section has to be built up by the steel section and the concrete section part using the corresponding input windows. Before starting composite calculations it is advisable to save the whole input data using the "File management window" (start-up window). From there the button "COMPOSITE" gives a link to the composite calculation window (Fig.4-43).


Fig.4-43 Composite calculation window (prcom.frm)
The name of the actual section and a small sketch of it are given together with the main dimensions. Input boxes have been provided to modify the safety-factors $\gamma_{\mathrm{a}}, \gamma_{\mathrm{c}}, \gamma_{\mathrm{r}}$, the concrete stress factor $\alpha_{c}$, the effective tension factor $f_{c t, e f f} / f_{c t m}$ and the $\beta$-value for the tension stiffening effect. Note: if $\mathrm{f}_{\mathrm{ct}, \mathrm{eft}} / \mathrm{f}_{\mathrm{ct}}=0$ no tension stiffening will be taken into consideration, what has been very useful to study the influence of concrete between the
cracks. Further the interacting vertical shear force can be defined, which fully will be assigned to the steel webs (concrete does not contribute to vertical shear). Before any calculation it will be checked automatically if the given shear value does not exceed the design shear resistance of all steel web elements together. Finally also the normal force of the whole composite cross section can be chosen, however for conventional beam calculations N has to be set equal to zero.

### 4.2.5.2 Theoretical background

The whole program is based on the clear separation of the composite cross section into its two basic section parts: steel and reinforced concrete. So with reference to the principle of equivalence the resulting inner forces and moments of the entire composite section can be set together by those of the section parts as shown in Fig.4-44. As the normal forces and bending moments of the steel and concrete section parts have already been related to a common datum line ( $y=0=$ steel bottom) they can simply be added without any further movement.


Fig.4-44 Inner forces and moments of section parts and entire composite section
Recall that the vertical shear force will be fully assigned to the web elements of the steel section. There it reduces their resistance with respect to additional normal stresses due to normal forces and bending moments.

Based on this theoretical background the following iterations can be carried out using the subroutines already described in detail in the previous chapters:

- $\mathrm{N}_{\mathrm{i}}-\mathrm{M}$-relation for a defined curvature
- $\mathrm{N}_{\mathrm{i}}-\mathrm{M}$-к-relation for a defined strain jump at the steel-concrete interface
- Given $M$ and strain jump at the interface (cross sectional design)
- Forces and moments to a given strain distribution
4.2.5.3 $\quad \mathrm{N}_{\mathrm{i}}-\mathrm{M}$-relation for a defined curvature (NMcomp)
plastic sagging

plastic hogging
X start with $\mathrm{Ni}=0$
X stop, when
A.steel= N.A.concr


## Calculation

Analogous to the iteration of the $\mathrm{N}_{\mathrm{ia}}-\mathrm{M}_{\mathrm{a}}$ '- or $\mathrm{N}_{\mathrm{ic}}-\mathrm{M}_{\mathrm{c}}$ '-curves the relation between the (internal) horizontal shear force in the dowel interface ( $\mathrm{N}_{\mathrm{i}}=\mathrm{N}_{\mathrm{ic}}$ ) and the entire bending moment of the composite section ( $\mathrm{M}^{\prime}$ ) can be derived. Assuming the standard case (entire normal force $\mathrm{N}=0$ ) the condition $\mathrm{N}_{\mathrm{ia}} \equiv \mathrm{N}_{\mathrm{ic}}$ must be fulfilled along the whole curve and $M^{\prime} \equiv M$. As $M^{\prime}=M_{a}{ }^{\prime}+M_{c}{ }^{\prime}\left(N_{i a} \equiv N_{i c}\right)$ the $N_{i}-M$-curve can simply be obtained by adding the $\mathrm{N}_{\mathrm{ia}}-\mathrm{M}_{\mathrm{a}}{ }^{\prime}$ - and the $\mathrm{N}_{\mathrm{ic}}-\mathrm{M}_{\mathrm{c}}{ }^{\prime}$-curves.


Fig.4-45 Determination of the relationship between $N_{i}$ and $M$
An input box has been arranged to prescribe a curvature $\kappa$. To ensure a result $\mathrm{N}_{\mathrm{i}} \neq 0$ the curvature $\kappa$ must be within the limits $-0.005 \% / \mathrm{m}$ and $+0.005 \% / \mathrm{m}$. Again the buttons „plastic sagging" and „plastic hogging" allow for a quasi-plastic calculation with $\kappa= \pm 10^{5} \% / \mathrm{m}$ and temporarily modified fully plastic stress-strain diagrams. The progress of the iteration is shown by a flood box and a comment line. Any calculation can be interrupted by pressing the "STOP"-button.

The determination of this $\mathrm{N}_{\mathrm{i}}-\mathrm{M}$ relationship is carried out within the global procedure "NMcomp" which is subdivided into the following six calculation steps:
1.) The procedure „steellimit" is used to determine the strain and force limits of the steel section part for the given curvature $\kappa$.
2.) The procedure "conclimit" is used to determine the strain and force limits of the concrete section part for the given curvature $\kappa$.
3.) In the third step the starting strains e 0 a and e 0 c of the section parts have to be calculated. The status of the option button "start with $\mathrm{N}_{\mathrm{i}}=0$ " (active or passive) determines the decisive starting point of the interaction curve in view of $\mathrm{N}_{\mathrm{i}}$.
3.1.) Conventionally („start with $\mathrm{N}_{\mathrm{i}}=0$ " is active) the interaction curve starts with $\mathrm{N}_{\mathrm{i}}=0$ (no horizontal shear force in dowel layer). Making use of the procedure „itNsteel" gives the corresponding steel strain e0a for $\mathrm{N}_{\mathrm{ia}}=\mathrm{N}$ (so normally $\mathrm{N}_{\mathrm{ia}}=0$ ). The concrete strain e 0 c for $\mathrm{N}_{\mathrm{ic}}=0$ will be iterated using the procedure "itNconcr".
3.2.) To analyse the full range of possible shear forces between the steel and concrete section parts also that part of the curve „before" reaching $\mathrm{N}_{\mathrm{i}}=0$ could be of interest. So if the option button „start with $\mathrm{N}_{\mathrm{i}}=0$ " is passive an additional calculation has to be carried out to evaluate the state of minimum shear connection: According to the definition that increasing interaction (at least at the beginning) means approach of the neutral axes of the section parts the state „before" reaching $\mathrm{N}_{\mathrm{i}}=0$ clearly can be defined where the neutral axes are even farer than at the state $N_{i}=0$. So based on the strains e0a and e0c a decisive starting value $\mathrm{N}_{\mathrm{i}, \text { start }}$ can be derived as shown in Fig.4-46 using the limiting normal forces of the section parts Nu1a, Nu2a, Nu1c and Nu2c (according to points 1.) and 2.)).


Fig.4-46 Derivation of the full range of shear connection
Based on that the decisive starting strain values of the steel and concrete section can be found: e0a results from the procedure „itNsteel" for $\mathrm{N}_{\mathrm{ia}}=\mathrm{N}_{\mathrm{i}, \text { start }}+\mathrm{N}$ and e0c results from the procedure „itNconcr" for $\mathrm{N}_{\mathrm{ic}}=\mathrm{N}_{\mathrm{i}, \text { start. }}$
4.) The next step concerns the end strain values of the section parts (e0ga, e0gc). Again it can be decided by an option button (active or passive status of "Stop, when N.A.steel=N.A.concr") whether the interaction curve should stop, where the neutral axes of steel and concrete are congruent, or if the full possible range of inner shear forces should be exhausted.
4.1.) For that case that the full possible range should be analysed (option button passive) the weaker section part determines the upper limit of the internal shear forces $N_{i, \text { end }}$. The following formulae can directly be derived from Fig.4-46, where it can be seen that the neutral axes of steel and concrete are approaching each other between the start of interaction and the congruence of the neutral axes and that they
are again separating beyond their congruence until the end of interaction is reached due to the limiting normal force of the weaker section part.

$$
\begin{array}{ll}
e 0 c>e 0 a & N_{i, e n d}=\min (N u 2 a-N, N u 1 c) \\
e 0 c<e 0 a & N_{i, e n d}=\max (N u 1 a-N, N u 2 c)
\end{array}
$$

The decisive end strain of the steel section e0ga then results from the procedure " $\mathrm{itNsteel}{ }^{\prime}$ for $\mathrm{N}_{\mathrm{ia}}=\mathrm{N}_{\mathrm{i}, \mathrm{end}}+\mathrm{N}$ and that of concrete e0gc results from the procedure „itNconcr" for $\mathrm{N}_{\mathrm{ic}}=\mathrm{N}_{\mathrm{i}, \text { end }}$.
4.2.) If the option button „stop when N.A.steel=N.A.concr" is active it will be checked first, if congruence of the neutral axes really would occur before reaching $\mathrm{N}_{\mathrm{i}, \text { end }}$ according to point 4.1.). So this stop-criterion only will become decisive if the following conditions are fulfilled:

$$
\begin{array}{lll}
(\mathrm{e} 0 \mathrm{a}<\mathrm{e} 0 \mathrm{c} & \text { and } & \mathrm{e} 0 \mathrm{ga}>\mathrm{e} 0 \mathrm{gc}) \\
(\mathrm{e} 0 \mathrm{a}>\mathrm{e} 0 \mathrm{c} & \text { and } & \mathrm{e} 0 \mathrm{ga}<\mathrm{e} 0 \mathrm{gc})
\end{array}
$$

Generally this will not occur for cross sections where the concrete plate completely is located above the steel section in combination with a relatively large curvature $\kappa$. However for slim-floors (where steel and concrete elements are sharing the same yrange) this criterion in any case will have to be taken into consideration.
The iteration for that decisive $N_{i}$, where the neutral axes of the steel and concrete section are congruent, then will be carried out according to the bisection method between the limits $N_{i, \text { start }}$ (point 3.)) and $N_{i, e n d}$ (point 4.1.)). At each iteration step of $N_{i}$ the corresponding normal force in the steel section can be calculated according to $\mathrm{N}_{\mathrm{ia}}=\mathrm{N}_{\mathrm{i}}+\mathrm{N}$ and that of the concrete section $\mathrm{N}_{\mathrm{ic}}=\mathrm{N}_{\mathrm{i}}$. Based on that the appropriate strains ep01a and ep01c can be determined with the procedures „itNsteel" and „itNconcr". The accuracy limits have been defined pursuant to a sensitivity study, as the iteration loops and therefore their deviations here are interconnected. The iteration will be stopped if the difference between the steel and concrete neutral axes falls short of an accuracy limit or if the iteration steps become smaller than a given tolerance. For the summary of prescribed tolerances reference is made to the end of chapter 4.2.


Fig.4-47 Full range of shear connection

In Fig.4-47 it is repeated that the full range of shear connection $N_{i}$ indeed may exceed the limits of $\mathrm{N}_{\mathrm{i}}=0$ and equality of neutral axes. Also beyond these limits the equilibrium condition $\mathrm{N}=\mathrm{N}_{\mathrm{i}}-\mathrm{N}_{\mathrm{ic}}$ is fulfilled for the composite cross section. Therefore
these parts of the interaction curve will not to be excluded in the global analysis. Their mechanical meaning and occurrence will have to be studied there.
The point of congruent neutral axes clearly separates the areas of approaching and separating tendency of the two neutral axes (steel and concrete). Therefore at this point the strain jump $\Delta \varepsilon$ in the shear interface changes its sign. As $\Delta \varepsilon$ gives the slope of the slip curve that directly is linked to a maximum or minimum of slip (see Fig.4-48).


Fig.4-48 Slip-curve

Fig.4-49 shows the described iteration to find that inner horizontal shear force $\mathrm{N}_{\mathrm{i}}$, where the neutral axes of the steel and concrete section parts are equal.


Sensitivity study for necessary provisional iteration tolerances
N.A. steel, concrete


Fig.4-49 Iteration of congruent neutral axes for the example section
5.) Knowing the limiting inner shear forces $N_{i, s t a r t}$ and $N_{i, \text { end }}$ this range is subdivided into 40 parts. Based on the procedure „itNsteel" the corresponding bending moment $\mathrm{M}_{\mathrm{a}}$ ' and the location of the neutral axis N.A.a are iterated for each $\mathrm{N}_{\mathrm{ia}}=\mathrm{N}_{\mathrm{i}}+\mathrm{N}$.
6.) For the same $N_{i}$-values ( $\mathbf{N}_{\mathrm{ic}}=\mathbf{N}_{\mathrm{i}}=\mathbf{N}_{\mathrm{ia}}-\mathbf{N}$ ) also the corresponding bending moment $\mathrm{M}_{\mathrm{c}}$ ' and N.A.c are determined with the routine „itNconcr". According to Fig.4-45 the bending moment of the entire composite section then can be set together by the moment of the steel and that of the concrete section ( $\mathbf{M}^{\prime}=\mathbf{M}_{\mathbf{a}}{ }^{\prime}+\mathbf{M}_{\mathbf{c}}{ }^{\prime}$ ) for the same inner shear force $N_{i} \quad\left(N=0 \Rightarrow M \equiv M^{\prime}\right)$. The cross section conditions are treated in the following hierarchic order: elastic, non-elastic, crushing of concrete, beyond $\varepsilon_{u}$. So if the stresses in both the steel and the concrete section are still within the elastic range for that interaction point "elastic" will be given. If "non-elastic" has been registered at least for one point the section will be treated as such kind and so on. Finally the error of the inner shear force after termination of the iteration will be reported: $\operatorname{dev}=\left|N_{i a}-N-N_{i c}\right|$

Fig.4-50 shows, how the $\mathrm{M}-\mathrm{N}_{\mathrm{i}}$-curve of the entire composite example section is set together adding up steel and concrete as described in Fig.4-45. It can clearly be seen that the range of possible inner shear forces is dominated by the force limits of the concrete plate being the weaker section part both in tension and compression. The isolated decisive part of shear interaction for the example section can be seen in Fig.4-51 for the given curvature $\kappa=10 \% / \mathrm{m}$.


Fig.4-50 Ni-M-interaction of example section (full range)


Fig.4-51 Ni-M-interaction of example section (detail)

Finally also the elastic and fully-plastic composite cross sectional properties are determined with the procedure "qswertecomp" according to the formulae given in Fig.4-52:

Attention has to be paid to the following fact: The formulae given in Fig.4-52 have been derived assuming that the concrete in tension (until $f_{c t, \text { eff }}$ is reached) behaves with the same elastic modulus as in compression. As in all iterations concrete stresses in tension have explicitly been neglected and have fully been assigned to the reinforcement (tension stiffening) the iteration results within the elastic range might differ slightly from those theoretical ones according to the formulae in Fig.4-52.

In the following the calculation results are described for the example section using $\kappa=10 \% / \mathrm{m}$ and $\mathrm{V}=0 \mathrm{kN}$. After termination of the iteration the composite calculation window is closed and the window for the presentation of the gained results is shown (Fig.4-53).
reference module $\quad E_{\text {ref }}=E_{\text {first steel element }}$
full fictitious area $A_{f}=\sum_{\text {steel , concrete, reinf. }} A_{i} \cdot \frac{E_{i}}{E_{\text {ref }}}$
ideal plastic normal force $N_{\text {pl,tension }}=\sum_{\text {steel }} A_{i} \cdot \frac{f_{y v}}{\gamma_{a}}+\sum_{\text {reinf. }} A_{i} \cdot \frac{f_{s k}}{\gamma_{r}}$
ideal plastic normal force $N_{\text {pl,compression }}=\sum_{\text {steel }} A_{i} \cdot \frac{f_{y v}}{\gamma_{a}}+\sum_{\text {reinf. }} A_{i} \cdot \frac{f_{s k}}{\gamma_{r}}+\sum_{\text {concrete }} A_{i} \cdot \frac{\alpha_{c} \cdot f_{c k}}{\gamma_{c}}$
plastic vertical shear force $V_{p l}=\sum_{\text {steel web elements }} A_{i} \cdot \frac{f_{y k, i}}{\gamma_{a}} \cdot \frac{1}{\sqrt{3}}$
elastic neutral axis $E N A=\frac{\sum_{\text {steel,concr,reinf. }} A_{i} \cdot y_{s, i} \cdot \frac{E_{i}}{A_{\text {ref }}}}{A_{f}}$
fictitious inertia $I_{f}=\sum_{\text {steel,concr. }} \frac{b_{i} \cdot h_{i}^{3}}{12} \cdot \frac{E_{i}}{E_{\text {ref }}}+\sum_{\text {steel,conor, ,einf. }} A_{i} \cdot\left(y_{s, i}-E N A\right)^{2} \cdot \frac{E_{i}}{E_{\text {ref }}}$ upper fict. res.moment $\quad W_{o}=\frac{I_{f}}{E N A}-y_{\max }$
lower fict. res.moment $W_{u}=\frac{I_{f}}{E N A-y_{\text {min }}}$
elastic curvature limit

elastic limit of bending $M_{e l}(N=0, V)=\kappa_{e l} \cdot E_{\text {ret }} \cdot I_{f}$

Fig.4-52 Formulae for the calculation of the composite sectional properties (qswertecomp)


|  |  reference E-modulus $=$ <br> Elastic Data: $210\left[\mathrm{kNimm} \mathrm{m}^{2}\right]$ |  |  |
| :---: | :---: | :---: | :---: |
| sec.prop. | E.N.A. = | 100.638 | [ mm ] |
|  | Area (ideell) | 15877 | [ mm 2 ] |
|  | Inertia (ideell) | 17641.5 | [cm4] |
|  | upper Resis.Moment | -907.7 | [cm3] |
|  | lower Resis.Moment = | 1753.0 | [cm3] |
|  | Elastic limit of bending (V)= | 257.3 | [ kNm ] |
|  | elastic limit of kapa (V)= | 6.95 | [\% $\%$ /m] |
|  | Ideal Plastic Data: |  |  |
|  | Normal force tens. (V) = | 4413.68 | $[\mathrm{k} N]$ |
|  | Normal force comp.(V) = | 5229.68 | [ kN ] |
|  | Shear force (webs) = | 797.79 | [ kN ] |

Fig.4-53 Window for the presentation of composite results (ep.frm)

It comprises the input data, an image of the composite section, the actual strains, stresses and forces (bare reinforcement in green, concrete action between the cracks in orange and shrinkage influence in brown) and a table and a graph of the calculated $\mathrm{N}_{\mathrm{i}}-\mathrm{M}$ '-interaction. There the final composite interaction curve is drawn with a thick line, the contribution of the steel section part is indicated by a thin blue and that of the reinforced concrete part by a thin green line. Those parts of the composite interaction curve drawn in light green represent the elastic range, the non-elastic parts are drawn in black, crushing of concrete is indicated with a red colour and those parts, where even $\varepsilon_{u}$ is exceeded in case of a too small reinforcement area, are drawn in white. Further for the horizontal $\mathrm{N}_{\mathrm{i}}$-axis also the corresponding $\eta$-values are given, representing the degree of shear connection. $\eta$ has been defined as the ratio between the actual inner shear force and the maximum possible shear force $\mathrm{N}_{\mathrm{i}, \text { end }}$ related to the weaker section part and the given curvature (acc. to chapter 4.2.5.3 4.1 $^{\text {I }}$ ):

$$
\eta=\frac{N_{i}}{N_{i, \text { end }}(\kappa)}
$$

So for ideal plastic stress-strain diagrams of each element $\mathrm{N}_{\mathrm{i}, \text { end }}(\kappa)$ is equal to the ideal plastic normal force of the weaker section part. Finally also the minimum and maximum bending moments are given in the diagram. The interaction table is combined with the possibility of a linear interpolation in view of the neutral axes N.A.a, N.A.c, $\mathrm{N}_{\mathrm{i}}$ and M'. Clicking on „sec.prop." switches to the window including the elastic and ideal (perfectly) plastic sectional properties calculated according to Fig.4-52.

To make the coherence between neutral axes, internal shear force and bending moment accessible to the user and therefore to get him the feeling where strengthening of the section parts would be economic or not special tools have been provided: Clicking on one row of the $\mathrm{N}_{\mathrm{i}}-\mathrm{M}$ '-table will produce the corresponding N.A.a and N.A.c within the section image, the strain, stress and force distribution and a reticule accentuating the actual location within the interaction curve. This graphical illustration will also be started after a linear interpolation or when following the interaction curve with the computer-mouse while the left mouse-button is pressed.

Using the button "Print results" will send the following four sheets to your printer: The first sheet contains a drawing of the actual composite section and the analysed range of strain distributions. At the bottom of this first sheet the $\mathrm{N}_{\mathrm{i}}-\mathrm{M}$ ' diagram will be displayed. The actual condition of the cross section in view of elasticity is indicated with different line styles. At the second sheet the interaction table will be given. The elastic and idealplastic cross-sectional properties are printed on sheet no. 3 and finally on sheet no. 4 you will find the full input data of the actual section.

To make the results also accessible to other programs they can be saved as a sequential data file with the button "Save results". The default file name is built up by that of the section (*.pro) and the extension „ep1" (elasto-plastic composite - first calculation block).

On the pages 212,213 and 214 you can find the print-outs of the example section for the full interaction range. The corresponding output file can be seen on page 215. The pages 216,217 and 218 give the results for the limited interaction range between $\mathrm{Ni}=0$ and the congruence of the neutral axes representing a detail of the full curve.



## Elasto-plastic results - composite section - kappa

Section:,"hat-shaped beam"
kappa [o/oo / m]=, 10
Savety factor steel:,1.1
Savety factor concrete:, 1.5
Stress factor concrete:,0.85
Savety factor reinf., ,1.15
Tension stiffening effect : fcteff/fctm $=, 1$
Tension stiffening effect: beta (short) $=, 0.4$
SAGGING moment
NO normal force in composite section (Nia=Nic)

$\begin{array}{cccc}\text { Table of reinforcem. elements (No,y,no,d,fy,E/1000,fu,eu,N(shrink.)) [ } N, m m] \\ 1 & 265 & 850 & 10\end{array}$
Ni-M Interaction Table (ya[mm],yc[mm],Ni[kN],M[kNm],eOa[o/oo],eOc[o/oo])
Ni start = ultimate , Ni limit = ultimate

| 43.66 | 4035.24 | -338.746 | 68.062 | 0.437 | 40.352 | non-elastic |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 45.16 | 580.70 | -301.809 | 76.076 | 0.452 | 5.807 | non-elastic |
| 46.67 | 429.32 | -264.096 | 84.173 | 0.467 | 4.293 | non-elastic |
| 48.18 | 398.90 | -225.858 | 92.298 | 0.482 | 3.989 | non-elastic |
| 49.61 | 337.71 | -189.046 | 100.043 | 0.496 | 3.377 | non-elastic |
| 51.05 | 287.90 | -151.795 | 107.863 | 0.511 | 2.879 | non-elastic |
| 52.48 | 278.92 | -114.454 | 115.823 | 0.525 | 2.789 | non-elastic |
| 53.91 | 275.68 | -77.113 | 123.686 | 0.539 | 2.757 | non-elastic |
| 55.35 | 272.29 | -39.771 | 131.562 | 0.554 | 2.723 | non-elastic |
| 56.78 | 268.71 | -2.430 | 139.452 | 0.568 | 2.687 | non-elastic |
| 58.22 | 264.74 | 34.912 | 147.358 | 0.582 | 2.647 | non-elastic |
| 59.65 | 255.18 | 72.253 | 155.442 | 0.597 | 2.552 | non-elastic |
| 61.08 | 247.01 | 109.595 | 163.430 | 0.611 | 2.470 | non-elastic |
| 62.52 | 239.77 | 146.936 | 171.319 | 0.625 | 2.398 | non-elastic |
| 63.95 | 233.19 | 184.277 | 179.112 | 0.640 | 2.332 | non-elastic |
| 65.39 | 227.13 | 221.619 | 186.811 | 0.654 | 2.271 | elastic |
| 66.82 | 221.47 | 258.960 | 194.420 | 0.668 | 2.215 | elastic |
| 68.25 | 216.14 | 296.302 | 201.943 | 0.683 | 2.161 | elastic |
| 69.69 | 211.02 | 333.643 | 209.408 | 0.697 | 2.110 | ic |
| 71.12 | 205.91 | 370.984 | 216.870 | 0.711 | 2.059 | elastic |
| 72.56 | 200.79 | 408.326 | 224.333 | 0.726 | 2.008 | castic |
| 73.99 | 195.68 | 445.667 | 231.795 | 0.740 | 1.957 | elastic |
| 75.42 | 190.56 | 483.009 | 239.258 | 0.754 | 1.906 | elastic |
| 76.86 | 185.45 | 520.350 | 246.720 | 0.769 | 1.855 | elastic |
| 78.29 | 180.34 | 557.691 | 254.183 | 0.783 | 1.803 | elastic |
| 79.73 | 175.22 | 595.033 | 261.646 | 0.797 | 1.752 | elastic |
| 81.16 | 170.11 | 632.374 | 269.108 | 0.812 | 1.701 | elastic |
| 82.59 | 164.99 | 669.716 | 276.571 | 0.826 | 1.650 | elastic |
| 84.03 | 159.88 | 707.057 | 284.033 | 0.840 | 1.599 | non-elastic |
| 85.46 | 154.61 | 744.398 | 291.456 | 0.855 | 1.546 | non-elastic |
| 86.90 | 149.03 | 781.740 | 298.800 | 0.869 | 1.490 | non-elastic |
| 88.33 | 143.05 | 819.081 | 306.066 | 0.883 | 1.431 | non-elastic |
| 89.76 | 136.58 | 856.423 | 313.252 | 0.898 | 1.366 | non-elastic |
| 91.20 | 129.49 | 893.764 | 320.358 | 0.912 | 1.295 | non-elastic |
| 92.63 | 121.54 | 931.105 | 327.386 | 0.926 | 1.215 | non-elastic |
| 94.07 | 112.31 | 968.447 | 334.348 | 0.941 | 1.123 | non-elastic |
| 95.50 | 100.87 | 1005.789 | 341.281 | 0.955 | 1.009 | non-elastic |
| 96.93 | 83.82 | 1043.130 | 348.371 | 0.969 | 0.838 | non-elastic |
| 98.37 | 54.54 | 1080.471 | 356.116 | 0.984 | 0.545 | non-elastic |
| 99.80 | -309.15 | 1117.813 | 363.888 | 0.998 | -3.092 | crush.of concr. |
| 101.22 | -9735.00 | 1154.746 | 371.574 | 1.012 | -97.350 | crush.of concr. |

max.deviation of Ni [kN], 0.00147
Section properties:
reference E-modulus [kN/mm²]:,210
E.N.A.from bottom line [mm]:,100.6376

Area (ideell) [mm2]:,15876.71
Inertia (ideell) [cm4]:,17641.47
upper Resis.Moment (ideell) [cm3]:,-907.659
lower Resis.Moment (ideell) [cm3]:,1752.97
El.limit of bending [kNm]:,257.3213
El.limit of kapa [o/oo/m]:,6.9458
Plastic normal force tension [kN]:,4413.682
Plastic normal force compr. [kN]:,5229.682, "without buckling-check"
Plastic shear force (web-parts): [kN],797.7931, "without buckling-check"
end



## Elasto-plastic results - composite section - kappa

Section:,"hhat-shaped beam
kappa [o/oo / m]=, 10
Savety factor steel:,1.1
Savety factor concrete:, 1.5
Stress factor concrete:,0.85
Savety factor reinf..,,1.15
Tension stiffening effect : fcteff/fctm $=, 1$
Tension stiffening effect: beta (short) $=, 0.4$
SAGGING moment
NO normal force in composite section (Nia=Nic)

| 1 | 50 | 0 | 250 | 15 | 355 | 210 | 510 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 175 | 15 | 10 | 200 | 380 | 210 | 450 |
| 3 | 185 | 205 | 45 | 10 | 380 | 210 | 450 |
| 11 | 300 | 0 | 250 | 15 | 355 | 210 | 510 |
| 12 | 415 | 15 | 10 | 200 | 380 | 210 | 450 |
| 13 | 370 | 205 | 45 | 10 | 380 | 210 | 450 |
| Table of concrete elements (No, x,y,w,h,fck,fctm) [N,mm] |  |  |  |  |  |  |  |
| 1 | 0 | 215 | 300 | 80 | 30 | 3 |  |
| 11 | 300 | 215 | 300 | 80 | 30 | 3 |  |
| Table of reinforcem. elements (No,y,no,d,fy,E/1000,fu,eu,N(shrink.)) [N,mm] |  |  |  |  |  |  |  |
| 1 | 265 | 8 | 10 | 550 | 200 | 620 | 0.1 |
| Ni-M Interaction Table (ya[mm],yc[mm],Ni[kN],M[kNm],e0a[o/oo],e0c[o/oo]) |  |  |  |  |  |  |  |

$\mathrm{Ni}-\mathrm{M}$ Interaction Table (ya[mm],yc[mm],Ni[kN],M[kNm],eOa[o/oo],eOc[o/oo])

| Ni start = 0, Ni limit = N.A. (steel=concrete |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 56.88 | 268.47 | 0.000 | 139.965 | 0.569 | 2.685 | non-elastic |
| 57.84 | 265.93 | 25.085 | 145.272 | 0.578 | 2.659 | non-elastic |
| 58.82 | 260.50 | 50.634 | 150.773 | 0.588 | 2.605 | non-elastic |
| 59.80 | 254.26 | 76.184 | 156.288 | 0.598 | 2.543 | non-elastic |
| 60.78 | 248.64 | 101.733 | 161.757 | 0.608 | 2.486 | non-elastic |
| 61.76 | 243.49 | 127.283 | 167.180 | 0.618 | 2.435 | non-elastic |
| 62.74 | 238.69 | 152.832 | 172.556 | 0.627 | 2.387 | non-elastic |
| 63.73 | 234.20 | 178.381 | 177.888 | 0.637 | 2.342 | non-elastic |
| 64.71 | 229.95 | 203.931 | 183.175 | 0.647 | 2.300 | elastic |
| 65.69 | 225.91 | 229.480 | 188.420 | 0.657 | 2.259 | elastic |
| 66.67 | 222.05 | 255.030 | 193.623 | 0.667 | 2.221 | elastic |
| 67.65 | 218.35 | 280.579 | 198.786 | 0.677 | 2.184 | elastic |
| 68.63 | 214.79 | 306.128 | 203.909 | 0.686 | 2.148 | elastic |
| 69.61 | 211.29 | 331.678 | 209.015 | 0.696 | 2.113 | elastic |
| 70.59 | 207.79 | 357.227 | 214.121 | 0.706 | 2.078 | elastic |
| 71.57 | 204.29 | 382.776 | 219.227 | 0.716 | 2.043 | elastic |
| 72.56 | 200.79 | 408.326 | 224.333 | 0.726 | 2.008 | elastic |
| 73.54 | 197.30 | 433.875 | 229.439 | 0.735 | 1.973 | elastic |
| 74.52 | 193.80 | 459.425 | 234.545 | 0.745 | 1.938 | elas |
| 75.50 | 190.30 | 484.974 | 239.651 | 0.755 | 1.903 | elastic |
| 76.48 | 186.80 | 510.523 | 244.757 | 0.765 | 1.868 |  |
| 77.46 | 183.30 | 536.073 | 249.863 | 0.775 | 1.833 | elastic |
| 78.44 | 179.80 | 561.622 | 254.968 | 0.784 | 1.798 | elastic |
| 79.42 | 176.30 | 587.171 | 260.074 | 0.794 | 1.763 | elastic |
| 80.33 | 173.07 | 610.756 | 264.788 | 0.803 | 1.731 | elastic |
| 81.31 | 169.57 | 636.305 | 269.894 | 0.813 | 1.696 | elastic |
| 82.29 | 166.07 | 661.854 | 275.000 | 0.823 | 1.661 | elastic |
| 83.27 | 162.57 | 687.404 | 280.105 | 0.833 | 1.626 | elastic |
| 84.25 | 159.07 | 712.953 | 285.210 | 0.843 |  | non-elastic |
| 85.24 | 155.47 | 738.503 | 290.289 | 0.852 | 1.555 | non-elastic |
| 86.22 | 151.72 | 764.052 | 295.331 | 0.862 | 1.517 | non-elastic |
| 87.20 | 147.80 | 789.601 | 300.337 | 0.872 | 1.478 | non-elastic |
| 88.18 | 143.70 | 815.151 | 305.306 | 0.882 | 1.437 | non-elastic |
| 89.16 | 139.37 | 840.700 | 310.236 | 0.892 | 1.394 | non-elastic |
| 90.14 | 134.78 | 866.250 | 315.130 | 0.901 | 1.348 | non-elastic |
| 91.12 | 129.88 | 891.799 | 319.985 | 0.911 | 1.299 | non-elastic |
| 92.10 | 124.59 | 917.348 | 324.805 | 0.921 | 1.246 | non-elastic |
| 93.08 | 118.79 | 942.897 | 329.590 | 0.931 | 1.188 | non-elastic |
| 94.07 | 112.31 | 968.447 | 334.348 | 0.941 | 1.123 | non-elastic |
| 95.05 | 104.83 | 993.996 | 339.090 | 0.951 | 1.048 | non-elastic |
| 96.02 | 95.71 | 1019.423 | 343.827 | 0.960 | 0.957 | non-elastic |

max.deviation of Ni [kN], 0.0009375
Section properties:
reference E-modulus [kN/mm²]:,210
E.N.A.from bottom line [mm]:,100.6376

Area (ideell) [mm2]:, 15876.71
Inertia (ideell) [cm4]:,17641.47
upper Resis.Moment (ideell) [cm3]:,-907.659
lower Resis.Moment (ideell) [cm3]:,1752.97
El.limit of bending [kNm]:,257.3213
El.limit of kapa [o/oo/m]:,6.9458
Plastic normal force tension [kN]:,4413.682
Plastic normal force compr. [kN]:,5229.682, "without buckling-check"
Plastic shear force (web-parts): [kN],797.7931, "without buckling-check" end

This first calculation block within the composite calculation window can also be used to determine the $\mathrm{M}_{\mathrm{Rd}}-\mathrm{V}_{\mathrm{Sd}}$ interaction curve of a composite section. In Fig.4-54 the maximum plastic sagging moment resistance of the example section has been evaluated for different values of interacting vertical shear. The decreasing moment resistance with increasing vertical shear comes from the fictitious reduction of the stress-strain diagram of the steel web elements due to the simultaneously acting shear stress.


Fig.4-54 Interaction between bending resistance and vertical shear
Apart from that the $\mathrm{N}_{\mathrm{i}}$-M-interaction curve is required for the derivation of the fullyplastic moment envelope according to the theory of partial shear connection [121][59,109,Annex E.3], which can be interpreted as the extension of the plastic analysis of frames to that of the shear interface. Excluding a stability failure and requiring a fully-plastic redistribution of horizontal shear forces along the shear interface the moment envelope e.g. for a single span beam can be gained directly from the plastic sagging Ni-M-interaction $(\kappa=+\infty)$ with two separate plastic neutral axes one for the steel part and one for the concrete part. Due to the redistribution of shear forces the available shear capacity at each beam location can be calculated as the sum of the shear capacities from the beam end to this point. So assuming e.g. an equidistant dowel spacing the shear design resistance $T_{\text {Rd }}[\mathrm{kN} / \mathrm{m}]$ will be constant along the beam axis and therefore the design shear force for each location $\ell$ can be calculated according to the formula:

$$
\mathrm{N}_{\mathrm{i}}=\sum_{\text {dowels }} \mathrm{N}_{\mathrm{i}, \mathrm{Rd}}=\mathrm{T}_{\mathrm{Rd}} \cdot \ell+\mathrm{N}_{\mathrm{Rd}, \text { end anchorage }}
$$

From that it follows that $\ell=\left(N_{i}-N_{\text {Rd,e.a. }}\right) / T_{\text {Rd }}$ what directly can be applied on the interaction diagram as shown in Fig.4-55 for the example section ( $\mathrm{V}=0$ ). There dowels providing an end-anchorage of $\mathrm{N}_{\mathrm{i}}=200 \mathrm{kN}$ have been supposed in combination with punched shear connectors along the whole beam providing $T_{\text {Rd }}=711 \mathrm{kN} / \mathrm{m}$. The final moment envelope is set together by the M - $\ell$-interaction curve depending on the beam span L. So the actual moment diagram must not exceed the plastic moment envelope $\left(\mathrm{M}_{\mathrm{sd}} \leq \mathrm{M}_{\mathrm{Rd}}\right)$ (Fig.4-56). Recall that this design method only is valid assuming a ductile shear interface allowing for plastic redistribution of horizontal shear forces.


Fig.4-55 Moment envelope gained form plastic sagging Ni-M-curve


Fig.4-56 Design ( $M_{S d} \leq M_{R d}$ ) for example section

The design of continuous composite beams is based on the same principles. In the following the calculation will be demonstrated for the example section for a symmetrical two-span beam loaded with eight single loads (Fig.4-57).


Fig.4-57 Two-span continuous beam
To be able to show the principles of calculation the shear capacity of the connector here will be assumed to be $T_{\text {Rd }}=100 \mathrm{kN} / \mathrm{m}$. Hence the shear capacity within the sagging moment area is $\mathrm{N}_{\mathrm{is}}=3[\mathrm{~m}]^{*} 100[\mathrm{kN} / \mathrm{m}]=300 \mathrm{kN}$. As shown in Fig.4-58 the corresponding partial bending moment resistance $\mathrm{M}_{\mathrm{s}}$ can be gained from the plastic sagging $\mathrm{N}_{\mathrm{i}}-\mathrm{M}$ diagram. The remaining shear capacity for activating the composite section in hogging is $N_{\text {ih }}=\left(8-2^{*} 3\right)[\mathrm{m}]^{*} 100[\mathrm{kN} / \mathrm{m}]=200 \mathrm{kN}$. The appropriate partial bending moment resistance $\mathrm{M}_{\mathrm{h}}$ is taken from the plastic hogging $\mathrm{N}_{\mathrm{i}}-\mathrm{M}$-diagram (Fig.4-58).

$K=-\infty$


Fig.4-58 Ni-M-diagram for plastic sagging and plastic hogging
Based on the theory of plastic design with plastic hinges in the sagging and hogging moment area the capable forces F can be calculated as:

$$
\mathrm{F}_{\mathrm{p}, \mathrm{Rd}}=\frac{\mathrm{Ms}+\mathrm{Mh} \cdot \frac{3}{8}}{4}=99.4 \mathrm{kN}
$$

Here it should be emphasised that according to the present state of EC4 $[59,109]$ partial shear connection ( $\eta<100 \%$ ) is not allowed within the hogging moment area in contradiction to the given example ( $\eta_{\text {hog }}=66 \%$ ).

As described before the plastic design of composite beams with ductile shear connectors is based on the knowledge about the design resistance $T_{R d}$ of the shear connectors. Going the opposite direction the resistance of the dowels can be gained not only from push-out- but also from four-point bending tests as shown in Fig.4-59. This evaluation method is given in [59,109,Annex E.2] for composite slabs but of course can also be applied to composite beams. It has successfully been used for the characterisation of punched shear connectors as described in detail in chapter 2.3.3.2.


Fig.4-59 Four-point bending test for shear resistance of connectors

To enable a test evaluation in view of the shear interface the other possible types of failure like bending and shear resistance of the whole cross section have to be excluded by an appropriate shear length $L_{s}$. From the test the bending moment resistance $M_{R, \text { test }}$ due to the applied forces and the dead load caused by failure of the connectors can be derived. The corresponding Ni-M-curve for the tested cross section has to be calculated for the measured geometrical and material properties for $\kappa=+\infty$. Fig. 4-60 shows the derivation of $T_{R, \text { test }}$ from $M_{R, t e s t . ~ O b v i o u s l y ~ f r o m ~ t h i s ~ k i n d ~ o f ~ t e s t s ~ o n l y ~}^{\text {. }}$ the plastic resistance value of the connectors can be determined in contrast to the much cheaper push-out tests, which even give the full force-slip curve. However it has become common practice to confirm the results from the push-out-tests by a final beam test.


Fig.4-60 Evaluation of four-point bending test
4.2.5.4 $\quad \mathrm{N}_{\mathrm{i}}-\mathrm{M}$-к-relation for a defined strain jump at the steel-concrete interface (DEcomp)


Calculation

This second calculation block within the composite calculation window is quite similar to the first one, however here the strain jump $\Delta \varepsilon$ is defined and the curvature $\kappa$ will be varied within given boundaries („from" - „to", 40 subdivisions). Ignoring the longer iteration time the solution for each $k$-value can be gained using the first calculation block and an interpolation for the given $\Delta \varepsilon$ (this has been done for check calculations). So the second calculation block is simply a further iteration loop within that the first calculation block is repeated.

As illustrated in Fig.4-44 $\Delta \varepsilon$ means the jump within the cross sectional strain distribution between the steel and the concrete section part at the shear interface. The integration of $\Delta \varepsilon$ along the beam axis leads to the slip, or vice versa $\Delta \varepsilon$ represents the slope of the slip-curve (Fig.4-48). $\Delta \varepsilon=0$ along the entire composite beam would mean an infinitely stiff shear-interaction without slip.

For the procedure „DEcomp" this strain jump has to be given as well as the vertical shear force $V$ and the normal force $N$ of the whole composite section. The curvature $\kappa$ then will be varied between the specified limits and the solution for each of these $\kappa$ values will be recorded for the final presentation of the $\kappa-\mathrm{N}_{\mathrm{i}}-\mathrm{M}$-curves. If $\kappa_{\text {from }}=\kappa_{\text {to }}$ the iteration will only be performed for this one specified $\kappa$-value. Further it has to be repeated that for $\kappa \approx 0$ no interaction between steel and concrete will be found and therefore $|\kappa|<0.005 \%$ 。 m will be modified to $\kappa=\kappa+0.01 \%$ 。 m .

The corresponding strain distribution, where $\Delta \varepsilon, \mathrm{V}$ and N are fulfilled for the specified curvature $\kappa$, is iterated in the following steps. The progress of the iteration is shown by a flood box and a comment line. The calculation can be interrupted by pressing the „STOP"-button.
1.) The procedure „steellimit" is used to determine the strain and force limits of the steel section part for the given curvature $\kappa$.
2.) The procedure "conclimit" is used to determine the strain and force limits of the concrete section part for the given curvature $\kappa$.
3.) In the third step the lower and upper limiting inner shear forces are calculated, where the weaker section parts reach their resistance. According to Fig.4-46 it follows that:

$$
\begin{aligned}
& \mathrm{N}_{\mathrm{i}, 1}=\max (\text { Nu1a }-\mathrm{N}, \mathrm{Nu} 2 \mathrm{c}) \\
& \mathrm{N}_{\mathrm{i}, 2}=\min (\mathrm{Nu} 2 \mathrm{a}-\mathrm{N}, \mathrm{Nu} 1 \mathrm{C})
\end{aligned}
$$

4.) Using the procedures „itNsteel" and „itNconcr" the corresponding datum strains then are determined for these force limits:

$$
\begin{aligned}
& \begin{array}{l}
\mathrm{N}_{\mathrm{ia}}=\mathrm{N}_{\mathrm{i}, 1}+\mathrm{N} \\
\mathrm{~N}_{\mathrm{ia}}=\mathrm{N}_{\mathrm{i}, 2}+\mathrm{N} \xrightarrow{\text { itNsteel }} \mathrm{eOa} \\
\mathrm{eOga}
\end{array} \\
& \begin{array}{l}
\mathrm{N}_{\mathrm{ic}}=\mathrm{N}_{\mathrm{i}, 1} \\
\mathrm{~N}_{\mathrm{ic}}=\mathrm{N}_{\mathrm{i}, 2}
\end{array} \xrightarrow{\mathrm{itNconcr}} \mathrm{eOc}
\end{aligned}
$$

5.) If one of the so-found strain limits of the steel section ( $\mathrm{e} 0 \mathrm{a}, \mathrm{e} 0 \mathrm{ga}$ ) reaches the yielding limits (e01ya or e02ya acc. to point 1.)) it has to be extended for this yieldplateau as shown at the bottom of Fig.4-61.


Fig.4-61 Consideration of the yielding plateau
The same philosophy has been applied to the concrete section. For that purpose the index „a" for steel would have to be changed to „c" for concrete in Fig.4-61. The possible range of $\Delta \varepsilon$ for the given curvature lies between the limits (ecmine-eamine) and (ecmaxe-eamaxe).
6.) If the given value of $\Delta \varepsilon$ lies in this possible range of $\Delta \varepsilon$ a solution can be found within the following iteration according to the bisection method:

- The datum steel strain e0a is iterated between eaminb and eamaxb
- For each e0a and the given $\kappa$ the normal force $N_{i a}$ in the steel section can be found with the procedure "steelstrain"
- $\mathrm{N}_{\mathrm{ic}}=\mathrm{N}_{\mathrm{ia}}-\mathrm{N}$ follows from the entire equilibrium. „itNconcr" is used to iterate the corresponding datum strain e 0 c to the given $\mathrm{N}_{\mathrm{ic}}$ and $\kappa$.
- The actual value of $\Delta \varepsilon$ is equal to (e0c-e0a) and will be compared with the given one. Then the iteration is continued with a new assumption for e 0 a until one of the following criteria is fulfilled: $\Delta \varepsilon$ reaches an appropriate accuracy, the change of e0a is smaller than a given tolerance or the stop-button is pressed.

Then these six calculation steps are repeated for the next $\kappa$-value. Fig.4-62 shows the described iteration of $\mathrm{e} 0 \mathrm{a}\left(=\varepsilon_{0, \text { steel }}\right)$ for the example section to fulfill $\Delta \varepsilon=0, \mathrm{~V}=0$ and $\mathrm{N}=0$ for $\kappa=10 \% / \mathrm{m}$.


Fig.4-62 Iteration of strain distribution to given $\Delta \varepsilon$

The results for the given range of curvature then are presented in the window shown in Fig.4-63. It comprises some input data, a sketch of the analysed composite cross section and three graphs and a table showing the coherence between $\kappa, N_{i}$ and $M$. The cross sectional condition is indicated by different colours: elastic in green, non-elastic in black, crushing of concrete in red and that part beyond $\varepsilon_{u}$ in white. In Fig.4-63 it can be seen very clearly the influence of the tension stiffening effect for $\kappa<0$, where the reinforcement is in tension. From the $М$-к-curve the actual secant bending stiffness can even be taken for the non-elastic parts according to the formula

$$
\mathrm{E} \cdot \mathrm{I}\left[\mathrm{kNm}^{2}\right]=\frac{\mathrm{M}[\mathrm{kNm}] \cdot 1000}{\kappa[\% / \mathrm{m}]}
$$

For the example section in the sagging moment area it can be calculated that $\mathrm{El}_{\mathrm{ini}}=240584 / 6.5=37013 \mathbf{k N m}^{2}$. This value can directly be compared with that given on page 214. Here differences might occur due to the different calculation philosophies in view of concrete in tension: For the elastic section properties concrete in tension is taken into consideration with the same Young's modulus as in compression whereas in all iterations the effect of concrete in tension fully has been assigned to the reinforcement (tension stiffening effect).

Finally there have been provided input boxes for linear interpolations between $\kappa, \mathrm{N}_{\mathrm{i}}$ and $M$. Furthermore also the recorded maximum errors of $N_{i}$ and $\Delta \varepsilon$ due to the iterations are given. Using the button „Print results" will send the following four sheets to your printer: The first sheet (see page 228) contains a drawing of the actual composite
section and the input data (safety factors, $\Delta \varepsilon, N, V$ ). At the bottom of this first sheet the $\mathrm{N}_{\mathrm{i}}-\mathrm{\kappa}$ diagram will be displayed. The drawing style of the curve depends on the status of the composite section in view of elasticity and is given in a legend. At the second sheet (page 229) you will find the $\mathrm{M}^{\prime}-\kappa_{-}$and the $\mathrm{M}^{\prime}-\mathrm{N}_{\mathrm{i}}-$ curve. Attention has to be paid that $\mathrm{M}=\mathrm{M}^{\prime}-\mathrm{N} \cdot \mathrm{e}$ for $\mathrm{N} \neq 0$ - or vice versa: for $\mathrm{N}=0$ : $\mathrm{M}=\mathrm{M}^{\prime}$. Sheet no. 3 (page 230) contains the $\kappa$ - $\mathrm{N}_{\mathrm{i}}-\mathrm{M}$ '-table and on sheet no. 4 you will find the full input data of the analysed section. To make the results also accessible to other programs they can be saved as a sequential data file with the button "Save results". The default file name is built up by that of the section (*.pro) and the extension „ep2" (elasto-plastic composite-second calculation block). The output file of the example calculation can be found on page 231.

Remark: Attention has to be paid to the following fact: The colour of each line segment depends on the section state (elastic, non-elastic,...) at its endpoint and therefore its accuracy depends on the curve resolution. So to get detailed information about the section state, especially for steep curves, it is necessary to refine the curve scale within the interesting area.


Fig.4-63 Window for the presentation of composite results (ep_de.frm)

## Elasto-plastic results - COMPOSITE

file $=c$ :Igeldynamicldiss\cobejolfiles\DISS.PRO hat-shaped beam



## kappa-Ni-M Table - COMPOSITE

file $=c:$ :Igeldynamicldiss\cobejolfiles\DISS.PRO

## hat-shaped beam

$$
\Delta \varepsilon=0.00 \quad[\%]
$$

max.deviation of $\mathrm{Ni}=0.001 \mathrm{kN} \quad$ max.deviation of delta epsilon $=0.005 \mathrm{o} / 00$

NO normal force NO vertical shear force
kappa [0/00]
$-10.00$
-9.50
-9.00
-8.50
$-8.00$
$-7.50$
$-7.00$
-6.50
-6.00
-5.50
-5.00
-4.50
$-4.00$
-3.50
-3.00
-2.50
$-2.00$
-1.50
-1.00
-0.50
0.01
0.50
1.00
1.50
2.00
2.50
3.00
3.50
4.00
4.50
5.00
5.50
6.00
6.50
7.00
7.50
8.00
8.50
9.00
9.50
10.00

Ni [kN]
-300.535
-291.959
-279.939
-267.496
-254.533
-242.486
-229.364
-217.683
-208.941
-204.562
-199.820
-195.089
-190.703
-185.965
-181.229
-176.489
-171.744
-167.370
-157.892
-82.053
57.958
116.308
168.810
227.146
285.489
343.828
396.329
454.676
513.006
571.346
626.777
682.198
740.534
798.876
851.382
895.136
933.049
933.049
968.063
994.309
1018.626

M [kNm]
$\begin{array}{ll}-201.952 & \text { ne } \\ -193.196 & \text { ne } \\ -183.725 & \text { ne }\end{array}$
-183.725 ne
$\begin{array}{ll}-174.165 & \text { ne } \\ -164.497 & \text { ne }\end{array}$
$\begin{array}{ll}-155.019 & \text { ne } \\ -145.318 & \text { ne }\end{array}$
$\begin{array}{ll}-135.917 & \text { ne } \\ -127.127 & \text { ne }\end{array}$
$\begin{array}{ll}-121.127 & \text { ne } \\ -119.246 & \text { ne } \\ -111.289 & \text { ne }\end{array}$
$\begin{aligned}-11.289 & \text { ne } \\ -103.334 & \text { ne } \\ -95.451 & \text { ne }\end{aligned}$
$\begin{array}{ll}-95.451 & \text { ne } \\ -87.495 & \text { ne }\end{array}$
$\begin{array}{ll}-79.539 & \text { ne } \\ -71.583 & \text { ne }\end{array}$
$\begin{array}{ll}-71.583 & \text { ne } \\ -63.625 & \text { ne } \\ -55.744 & \text { ne }\end{array}$
$\begin{array}{rc}-55.744 & \text { ne } \\ -46.801 & \text { ne } \\ -24.047 & \text { el }\end{array}$
$\begin{array}{rl}-24.047 & \mathrm{el} \\ 0.061 & \mathrm{el} \\ 18.719 & \mathrm{el}\end{array}$
$\begin{array}{ll}18.719 & \text { el } \\ 37.517 & \text { el } \\ 55.146 & \text { el } \\ 73.940 & \text { el }\end{array}$
$\begin{array}{rr}73.940 & \text { el } \\ 92.737 & \text { el } \\ 111.532 & \text { el }\end{array}$
$\begin{array}{ll}111.532 & \text { el } \\ 129.160 & \text { el }\end{array}$
$\begin{array}{ll}129.160 & \text { el } \\ 147.957 & \text { el } \\ 166.751 & \text { el }\end{array}$
$\begin{array}{ll}166.751 & \text { el } \\ 185.546 & \text { el } \\ 203.761 & \text { el }\end{array}$
$\begin{array}{ll}203.761 & \text { el } \\ 221.973 & \text { el }\end{array}$
240.767 el
$\begin{array}{ll}259.561 & \text { ne } \\ 276.983 & \text { ne }\end{array}$
$\begin{array}{ll}292.983 & \text { ne } \\ 306.655 & \text { ne } \\ & \text { ne }\end{array}$
$\begin{array}{ll}306.655 & \text { ne } \\ 320.219 & \text { ne }\end{array}$
332.120 ne
343.678
el=elastic ne=non-elastic cr=crushing of concrete **=epsilon > epsilon ultimate
$\mathrm{Nic}=\mathrm{Ni}$ (Normal force in concrete part; compression $>\mathbf{0}$ ) $\mathbf{N i a}=\mathbf{N i}$ (Normal force in steel part; tension >0)

Ni (Shear force in dowel layer)

## Elasto-plastic results - composite section - delta epsilon

Section:, "hat-shaped beam"
DELTA EPSILON [0/oo]:, 0
Savety factor steel:, 1.1
Savety factor concrete:, 1.5
Stress factor concrete:, 0.85
Savety factor reinf.., 1.15
Tension stiffening effect : fcteff/fctm =, 1
Tension stiffening effect: beta (short) $=, 0.4$
NO normal force in composite section (Nia=Nic)
Table of steel elements ( $\mathrm{No}, \mathbf{x}, \mathbf{y}, \mathbf{w}, \mathrm{h}, \mathrm{fyk}, \mathrm{E} / 1000, \mathrm{fu}, \mathrm{eu}, \mathrm{flange}=0 / \mathrm{web}=1$ ) [ $\mathrm{N}, \mathrm{mm}$ ]

| 1 | 50 | 0 | 250 | 15 | 355 | 210 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 175 | 15 | 10 | 200 | 380 | 210 |
| 3 | 185 | 205 | 45 | 10 | 380 | 210 |
| 11 | 300 | 0 | 250 | 15 | 355 | 210 |
| 12 | 415 | 15 | 10 | 200 | 380 | 210 |
| 13 | 370 | 205 | 45 | 10 | 380 | 210 |
| Table of concrete elements (No,x,y,w,h,fck,fctm) |  |  |  |  |  |  |
| 1 | 0 | 215 | 300 | 80 | 30 | 3 |
| 11 | 300 | 215 | 300 | 80 | 30 | 3 |

Table of reinforcem. elements (No,y,no,d,fy,E/1000,fu,eu,N(shrink.)) [ $\mathrm{N}, \mathrm{mm}$ ]

| 1 | 265 | 8 | 10 | 550 | 200 | 620 | 0.1 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

kappa-Ni-M Table (kappa[o/oo / m],Ni[kN],M[kNm])
$\begin{array}{llll}-10.0 & -300.54 & -201.95 & \text { non-elastic }\end{array}$
$-9.5 \quad-291.96$-193.20 non-elastic
$\begin{array}{llll}-9.0 & -279.94 & -183.72 & \text { non-elastic }\end{array}$
-8.5 $\quad$-267.50 -174.16 non-elastic
-8.0 $\quad-254.53 \quad-164.50$ non-elastic
$\begin{array}{lll}-7.5 & -242.49 & -155.02 \\ \text { non-elastic }\end{array}$
$\begin{array}{llll}-7.0 & -229.36 & -145.32 & \text { non-elastic }\end{array}$
-6.5 -217.68 -135.92 non-elastic
$-6.0 \quad-208.94 \quad-127.13$ non-elastic
$\begin{array}{llll}-5.5 & -204.56 & -119.25 & \text { non-elastic }\end{array}$
-5.0 $\quad$-199.82 -111.29 non-elastic
$-4.5 ~-195.09 \quad-103.33$ non-elastic
-4.0 $\quad$-190.70 -95.45 non-elastic
$\begin{array}{llll}-3.5 & -185.96 & -87.49 & \text { non-elastic }\end{array}$
$\begin{array}{llll}-3.0 & -181.23 & -79.54 & \text { non-elastic }\end{array}$
$\begin{array}{llll}-2.5 & -176.49 & -71.58 & \text { non-elastic }\end{array}$
$\begin{array}{llll}-2.0 & -171.74 & -63.62 & \text { non-elastic }\end{array}$
$\begin{array}{llll}-1.5 & -167.37 & -55.74 & \text { non-elastic }\end{array}$
$\begin{array}{llll}-1.0 & -157.89 & -46.80 & \text { non-elastic }\end{array}$
$\begin{array}{lll}-0.5 & -82.05 & -24.05 \\ \text { elastic }\end{array}$
$\begin{array}{lll}0.0 & -0.38 & 0.06 \text { elastic }\end{array}$
elastic
elastic
elastic
elastic
elastic
elastic
elastic
elastic
elastic
elastic
elastic
$\begin{array}{llll}5.5 & 626.78 & 2031.76 & \text { elastic }\end{array}$
$\begin{array}{lll}6.5 & 740.53 & 240.77 \\ \text { elastic }\end{array}$
$\begin{array}{llll}7.0 & 798.88 & 259.56 & \text { non-elastic }\end{array}$
$\begin{array}{lll}7.5 & 851.38 & 276.98 \\ \text { non-elastic }\end{array}$
$\begin{array}{llll}8.0 & 895.14 & 292.45 & \text { non-elastic }\end{array}$
$8.5 \quad 933.05 \quad 306.66$ non-elastic
$\begin{array}{llll}9.0 & 968.06 & 320.22 & \text { non-elastic } \\ 9.5 & 994.31 & 332.12 & \text { non-elastic }\end{array}$
$\begin{array}{llll}9.5 & 994.31 & 332.12 & \text { non-elastic }\end{array}$
$10.0 \quad 1018.63 \quad 343.68$ non-elastic
max.deviation of $\mathrm{Ni}[\mathrm{kN}], 0.00149$
max.deviation of delta eps [o/oo], 0.00468
end

### 4.2.5.5 Given M and strain jump at the interface (cross sectional design) (QSNWcomp)



Additionally to $\Delta \varepsilon, \mathrm{V}$ and N in this third calculation block also the overall bending moment M has to be defined. Within the iterative procedure "QSNWcomp" the corresponding strain distribution ( $\kappa, \varepsilon_{0 a}, \varepsilon_{0 c}$ ) will be determined. Strictly this could also be done by analysing the corresponding range of $\kappa$ with the second calculation block and making an interpolation for the searched M . So this calculation block is built up by the second one (三 procedure „QSNWcomp_sub") repeated within a further additional iteration loop, where $\kappa$ is varied. The calculation can be subdivided into the following four steps:
1.) Any starting value for the curvature $\kappa$ has to be determined, where a solution for the given $\Delta \varepsilon, \mathrm{V}$ and N can be found. Obviously the iteration can drastically be speeded up by giving an approximated $\kappa_{\text {start }}$, known from the global analysis or from the second calculation block. If no value is given for $\kappa$ it will be set $+0.1 \%$ for $\Delta \varepsilon>0$ or $-0.1 \%$ for $\Delta \varepsilon<0$.
2.) Using the procedure „QSNWcomp_sub" it will be checked, if a solution can be found for $\kappa_{\text {start }}$ fulfilling $\Delta \varepsilon, \mathrm{V}$ and N . There are three possibilities:
2.1.) The corresponding moment $\mathrm{M}_{\text {start }}$ to the given curvature $\kappa_{\text {start }}$ approximately is equal to the given value M and therefore the solution is already found.
2.2.) It is possible to find a solution for $\kappa_{\text {start }}$ but $\mathrm{M}_{\text {star }} \neq \mathrm{M}$ and therefore the calculation has to be continued with step 3.
2.3.) No solution can be found for the estimated $\kappa_{\text {start }}$ and therefore a new value has to be assumed. Step by step the starting curvature will be modified in the following way until a starting solution can be found:
$\kappa_{\text {start }}=\kappa_{\text {start }} \pm(0.2,0.4,0.6,0.8,1.0,2,4,6,8,10,20,40,60,80,100,200,400,600,800,1000) \% / \mathrm{m}$
3.) Based on $\kappa_{\text {start }}$ any curvature $\kappa_{\text {end }}$ at the opposite side of the solution has to be determined. For that purpose $\kappa$-values are examined using the procedure "QSNWcomp_sub" (step by step more far away from $\kappa_{\text {start }}$ ) until a curvature is found, which fulfils the following conditions:
a) a solution can be found meeting the given $\Delta \varepsilon, \mathrm{V}$ and N
b) the corresponding $M_{\text {end }}$ has to lie at the opposite side of $M_{\text {start }}$ in view of the given bending moment M , because as the relation of $\mathrm{M}-\kappa$ is continuous and monotous
the final solution in that case then can be found in between $\kappa_{\text {start }}$ and $\kappa_{\text {end }}$ by an iteration

$$
\begin{array}{lll}
\text { if } M_{\text {start }}<M & \Rightarrow M_{\text {end }}>!M \\
\text { if } M_{\text {start }}>M & \Rightarrow & M_{\text {end }}<!M
\end{array}
$$

Step by step $\kappa_{\text {end }}$ is iterated in the following way:

$$
\kappa_{\text {end }}=\kappa_{\text {start }} \underset{\text { if } M_{\text {start }}>M}{ \pm} \Delta \kappa=(0.1,1,10,100,10,1,0.1,0.01) \% / \mathrm{m}
$$

If the condition a) cannot be fulfilled a new $\Delta \kappa$ will be tried. If the condition b) cannot be fulfilled $\kappa_{\text {start }}$ will be set equal to the so-found $\kappa_{\text {end }}$ and based on that a new $\kappa_{\text {end }}$ will be determined with the next ( ${ }^{*}$ ) or same ( $\left.{ }^{* *}\right) \Delta \kappa$.
4.) Finally between the so-gained limits $\kappa_{\text {start }}$ and $\kappa_{\text {end }}$ that special curvature $\kappa$ will be iterated according to the bisection method, which meets $\Delta \varepsilon, V, N$ and $M$. The iteration process will be indicated by a flood box and a comment line. It will be terminated if one of the following criteria becomes decisive: M has been reached with an appropriate accuracy, the changes of $\kappa$ are smaller than a given tolerance or the STOP-button is pressed.
The iteration curve for the example section looking for $\kappa$ fulfilling $\Delta \varepsilon=0, \mathrm{~V}=0, \mathrm{~N}=0$ and $M=343.69 \mathrm{kN}$ can be seen in Fig.4-64, which could be gained applying the second calculation block.


Fig.4-64 Iteration of curvature $\kappa$ to given $M$ and $\Delta \varepsilon$

The final result consisting of $\kappa, \varepsilon_{0 a}$ and $\varepsilon_{0 c}$ then is handed over to the fourth calculation block, which will be described in the following chapter.
4.2.5.6 Forces and moments to a given strain distribution


In contrast to the three calculation blocks described before this fourth one does not consist of interlaced iteration loops but calculates the inner forces and moments based on a given linear strain distribution ( $\kappa, \varepsilon_{0 a}$ or $y_{\text {steel }}, \varepsilon_{0 c}$ or $y_{\text {concr }}$ as shown in Fig.4-65) and the material laws of the several steel, concrete and reinforcement elements. Of course the remaining properties in view of longitudinal stresses depend on the existence of a vertical shear force V and the safety factors.


Fig.4-65 Linear strain distribution
This calculation is directly based on the basic stress-strain relations and does not call for any iterative solutions. Therefore it proved to be very useful for checks of the several iteration results. Further use was made of it to analyse several cross sections of test specimens, where the strain distribution has been measured with strain gauges.

The calculation can be subdivided into the following three steps:
1.) The inner normal force of the steel section $N_{i a}$, the bending moment $M_{a}$ ' and the status in view of elasiticity are determined with the procedure "steelstrain" based on the given $\kappa$ and $\varepsilon_{0 a}$.
2.) The inner normal force of the concrete section $N_{i c}$, the bending moment $M_{c}$ ' and the status in view of elasiticity are determined with the procedure „concstrain" based on the given $\kappa$ and $\varepsilon_{0 c}$.
3.) Finally the composite section is set together adding up the section parts steel and concrete based on the principles given in Fig.4-44.


Fig.4-66 Window for the presentation of composite results (ep_qsnw.frm)
The results then are presented within the window shown in Fig.4-66. It contains the input data ( $\kappa, \varepsilon_{0 \mathrm{a}}, \varepsilon_{0 \mathrm{c}}$ ), from which $\Delta \varepsilon$ and the neutral axes of steel and concrete are derived. Further a sketch of the analysed composite section including the neutral axes, the chosen strain distribution and the resulting stress distribution are given. Pressing
the left mouse button and moving the mouse within the section sketch, the strain or the stress graphs will produce a reticule and shows the corresponding $x-, y-, \varepsilon$ - and $\sigma$-values. This enables then to pick special values from the graphs. In the centre of the presentation window you can find a table, where the strains and stresses of each element are listed. Concerning the reinforcement the resulting force explicitly is set together by the parts of bare steel, contribution of the concrete between the cracks and shrinkage. Clicking on an element within the table will high-lighten it in the section graphic as well as in the strain and stress distribution. At the bottom the inner normal forces and bending moments of the section parts are given as well as those of the entire composite section set together by steel and concrete. The arrows of the forces and moments automatically are drawn corresponding to their sign pointing to the effective direction. The sketch of the composite section set together by an I-shaped steel beam and an upper concrete plate only is symbolical.

With the „Print"-button the following three sheets can be printed: The first sheet comprises a sketch of the composite section, the input data ( $\kappa, \varepsilon_{0 a}, \varepsilon_{0 c}$ ) and the inner forces and moments of the section parts as well as those of the entire composite section (see page 237). On the second sheet (page 238) the strain and stress distribution is given both as graphs and as a table. The third sheet again gives the whole input data of the composite cross section. To make the results also accessible to other programs they can be saved as a sequential data file with the button "Save results". The default file name is built up by that of the section (*. pro) and the extension „ep3" (elasto-plastic composite - third calculation block). The output file of the example calculation can be found on page 239.


Internal forces - Steel and concrete parts separately


Mc'= 262.23 kNm
$\mathrm{Nc}=1018.70 \mathrm{kN}$

Internal forces - Composite cross section



## Composite cross section check

Section:
kappa $[\mathrm{o} / \mathrm{oo} / \mathrm{m}]=$
hat-shaped beam
EPS steel $(\mathrm{y}=0)$ [ $[0 / 00]=$
10
EPS concr ( $\mathrm{y}=0$ ) [ $\mathrm{o} / \mathrm{oo}]=$
0.95996
delta eps [0/00]= 0.95996

Neutral axis steel [mm]=
0
Neutral axis concr [mm]= 96

Savety factor steel: 96

Savety factor concrete:
Stress factor concrete:
1.5

Savety factor reinf.:
. 85
Tension stiffening effect : fcteff/fctm =
Tension stiffening effect: beta (short) $=$ 0.4

Vertical shear force in steel part $[\mathrm{kN}]=$
0
Normal force in steel part (tension $>0$ ) $[\mathrm{kN}]=$
1018.711

Normal force in concr part (compr. $>0$ ) $[\mathrm{kN}]=$
1018.71

Normal force in composite (tension>0) [kN]= 0.0005

Horizontal shear force in dowel layer $[\mathrm{kN}]=$
1018.71

Bend.moment in steel part (at $\mathrm{y}=0$ ) $(\mathrm{sagg}>0)[\mathrm{kNm}]=$ 81.46352

Bend.moment in concr part (at $\mathrm{y}=0$ ) (sagg>0) $[\mathrm{kNm}]=$ 262.23 Bend.moment in composite (at $\mathrm{y}=0$ ) (sagg>0) [kNm]= 343.6935 SAGGING area
NON-elastic parts
Table of steel elements ( $\mathrm{No}, \mathbf{x}, \mathrm{y}, \mathrm{w}, \mathrm{h}, \mathrm{fyk}, \mathrm{E} / 1000$, fu,eu,flange $=\mathbf{0} / \mathrm{web}=1$ ) [ $\mathrm{N}, \mathrm{mm}$ ]

| 1 | 50 | 0 | 250 | 15 | 355 | 210 | 510 | 0.1 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 175 | 15 | 10 | 200 | 380 | 210 | 450 | 0.1 | 1 |
| 3 | 185 | 205 | 45 | 10 | 380 | 210 | 450 | 0.1 | 0 |
| 11 | 300 | 0 | 250 | 15 | 355 | 210 | 510 | 0.1 | 0 |
| 12 | 415 | 15 | 10 | 200 | 380 | 210 | 450 | 0.1 | 1 |
| 13 | 370 | 205 | 45 | 10 | 380 | 210 | 450 | 0.1 | 0 |
| Table of concrete elements (No, $\mathrm{x}, \mathrm{y}, \mathrm{w}, \mathrm{h}, \mathrm{fck}, \mathrm{fctm}$ ) [ $\mathrm{N}, \mathrm{mm}]$ |  |  |  |  |  |  |  |  |  |
| 1 | 0 | 215 | 300 | 80 | 30 | 3 |  |  |  |
| 11 | 300 | 215 | 300 | 80 |  | 3 |  |  |  |
| Table of reinforcem. elements (No,y,no,d,fy,E/1000,fu,eu,N(shrink.)) [N,mm] |  |  |  |  |  |  |  |  |  |
| 1 | 265 | 8 | 10 | 550 | 200 | 620 | 0.1 | 0 |  |

Table of strains and stresses
steel elements (No,eps bottom,eps top,sigma bottom,sigma top,0=el/1=non-el) (eps[o/oo],sigma[N/mm2])

| 1 | 0.95996 | 0.80996 | 201.5916 | 170.0916 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 0.80996 | -1.19004 | 170.0916 | -249.9084 | 0 |
| 3 | -1.09004 | -1.19004 | -228.9084 | -249.9084 | 0 |
| 11 | 0.95996 | 0.80996 | 201.5916 | 170.0916 | 0 |
| 12 | 0.80996 | -1.19004 | 170.0916 | -249.9084 | 0 |
| 13 | -1.09004 | -1.19004 | -228.9084 | -249.9084 | 0 |

concrete elements (No,eps bottom,eps top,sigma bottom,sigma top,0=el/1=non-el/2=crush) (eps[0/oo],sigma[N/mm2])

| 1 | -1.19004 | -1.99004 | -14.98569 | -17 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 11 | -1.19004 | -1.99004 | -14.98569 | -17 | 1 |

reinforcement elements (No,eps,Fsteel,Fconcr,Fshrink,sum(F)) (eps[0/00],F[kN])

| 1 | -1.69004 | -212.3767 | 0 | 0 | -212.3767 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |

end

The described calculation has also been used to study the influence of the tension stiffening effect on the course of the internal horizontal shear force $N_{i}$ and the entire bending moment M along the whole range of possible curvatures $\kappa$ for the example section, prescribing $\Delta \varepsilon=0, \mathrm{~V}=0$ and $\mathrm{N}=0$ (see Fig.4-67).





Fig.4-67 Influence of the tension stiffening effect

Furthermore this calculation block can directly be used to determine the N-M interaction curve for the design of composite columns:
For that purpose $\kappa$ has to be set equal to $+\infty 0$ and then $N$ and $M$ ' have to be determined for a varying location of neutral axes $y_{\text {stee }}=y_{\text {concrete }}$. The common y values should start with zero and end, where $\mathrm{N}=0$ (due to the different behaviour of concrete in tension and compression this occurs for $\mathrm{y}>\mathrm{h}_{\mathrm{c}} / 2$ ). Each M' (related to the bottom datum line) then has to be converted to the centre line ( $y=h_{c} / 2$ ) using the formula

$$
=M^{\prime}+N \cdot \frac{h_{c}}{2}
$$

In Fig.4-68 this calculation has been done for the example section defined in [101]. The comparison between the curves shows perfect agreement.


| $\mathbf{y}$ | N | $\mathrm{M}^{\prime}$ |
| :---: | :---: | :---: |
| 0 | -7434 | 1115 |
| 5 | -6453 | 1113 |
| 10 | -5471 | 1105 |
| 15 | -4489 | 1093 |
| 19 | -3704 | 1080 |
| 25 | -3500 | 1075 |
| 40 | -3217 | 1066 |
| 70 | -1951 | 998 |
| 100 | -1539 | 963 |
| 150 | -851 | 877 |
| 200 | -163 | 756 |
| 212 | -2 | 722 |




Fig.4-68 N-M interaction curve for composite column design

### 4.2.5.7 Summary

Within the composite calculation window there are four different types of cross section analyses. The first calculation block is used to determine the relation between inner horizontal shear force and bending moment for a given curvature. This calculation is repeated within a further iteration loop for the second calculation block, where a given $\Delta \varepsilon$ shall be met for a variety of $\kappa$-values. And finally for the third iteration block a further iteration loop has been added to find that special strain distribution, where both $\Delta \varepsilon$ and a given bending moment M are met. The fourth block represents the bare calculation of stresses from a given strain distribution, finally leading to inner forces and moments by integrating the stresses over the whole section. In Fig.4-69 the convergence and stop criteria of all used iterations are summarised. They have been defined as a compromise between accuracy and iteration time. For those cases of interlaced iteration loops sensitivity studies have been carried out. All these considerations serve as the
fundamental background for the global analysis, where finite beam elements and joints are combined one element behind the other forming a single span, continuous or semicontinuous beam.

| Iteration-procedure |  | Convergence criterion | Stop-criterion |
| :---: | :---: | :---: | :---: |
| itNsteel (genau) |  | $\left\|\mathrm{N}-\mathrm{N}_{\mathrm{ia}}\right\| \leq \max ($ genau, $1[\mathrm{~N}]$ ) interpolation if $\Delta \varepsilon_{0 \mathrm{a}}<10^{-5} \%$ 。 | iteration time $>30 \mathrm{sec}$. |
| Nsteeliteration |  | genau $=\Delta \mathrm{N}_{\mathrm{a}} /$ accur1 resp. given value |  |
| NMsteel |  | $\begin{aligned} & \mathrm{N}_{\mathrm{ia}}=0 \rightarrow \text { genau }=\Delta \mathrm{N}_{\mathrm{a}} / \text { accur } 1 \\ & \text { else } \rightarrow \text { genau }=\Delta \mathrm{N}_{\mathrm{a}} / \text { accur } 2 \end{aligned}$ |  |
| itNconcr (genau) |  | $\left\|\mathrm{N}-\mathrm{N}_{\text {ic }}\right\| \leq \max$ (genau, 1 [N]) interpolation if $\Delta \varepsilon_{0 c}<10^{-5} \%$ 。 | iteration time > 30 sec . |
| Nconcriteration |  | genau $=\Delta \mathrm{N}_{\mathrm{c}}$ /accur1 resp. given value |  |
| NMconcr |  | $\begin{aligned} & \mathrm{N}_{\mathrm{ic}}=0 \rightarrow \text { genau }=\Delta \mathrm{N}_{\mathrm{C}} / \text { accur } 1 \\ & \text { else } \rightarrow \text { genau }=\Delta \mathrm{N}_{\mathrm{c}} / \text { accur2 } \end{aligned}$ | $\leqslant$ |
| NMcomp | $N_{i, \text { start }}, N_{i, \text { end }}$ | genau $=\Delta \mathrm{N}_{\mathrm{a}} /$ accur1 resp. $\Delta \mathrm{N}_{\mathrm{c}} /$ accur1 | STOP-button |
|  | N.A.a $=$ N.A.c ${ }_{\text {c }}$ | $\begin{gathered} \text { genau }=\left\|N_{i, \text { start }}-N_{i, \text { end }}\right\| / \text { accur1 } \\ \mid \text { N.A.a }- \text { N.A.c } \mid<\max \left(\mathrm{h}_{\text {ges }} / 1000 / \mathrm{pf}, 0.5 / \mathrm{pf}[\mathrm{~mm}], \mathbf{0 . 0 0 1 [ m m ] )}\right. \end{gathered}$ | $$ |
|  | steel part | $\begin{aligned} \mathrm{N}_{\mathrm{ia}}=\mathrm{N}_{\mathrm{i}, \text { start }} \text { or } \mathrm{N}_{\mathrm{i}, \text { end }} & \rightarrow \text { genau }=\left\|\mathrm{N}_{\mathrm{i}, \text { start }}-\mathrm{N}_{\mathrm{i}, \text { end }}\right\| / \text { accur1 } \\ \text { else } & \rightarrow \text { genau }=\left\|N_{\mathrm{i}, \text { start }}-\mathrm{N}_{\mathrm{i}, \text { end }}\right\| / \text { accur2 } \end{aligned}$ | STOP-button |
|  | concrete part | genau $=\left\|\mathrm{N}_{\mathrm{i}, \text { start }}-\mathrm{N}_{\mathrm{i}, \text { end }}\right\| /$ accur1 | STOP-button |
| DEcomp, QSNWcomp_sub | $\mathrm{N}_{\mathrm{i}, 1}, \mathrm{~N}_{\mathrm{i}, 2}$ | genau $=\Delta N_{a} /$ accur1 resp. $\Delta N_{c} /$ accur1 |  |
|  | $\Delta \varepsilon$ | $\begin{gathered} \text { genau }=\left\|N_{\mathrm{i}, 1}-\mathrm{N}_{\mathrm{i}, 2}\right\| / \text { accur1 } \\ \left\|\Delta \varepsilon-\Delta \varepsilon_{\text {iter. }}\right\|<\max (\|\Delta \varepsilon\| / 1000 / \mathrm{pf}, 0.005 / \mathrm{pf}[\%], \mathbf{0 . 0 0 0 1}[\% \mathrm{\%}]) \end{gathered}$ | STOP-button $\Delta \varepsilon_{0 \mathrm{a}}<10^{-10} \%$ |
| QSNWcomp |  | $\left\|\mathrm{M}-\mathrm{M}_{\text {iter. }}\right\|<\max (\|\mathrm{M}\| / 1000 / \mathrm{pf}, 0.01 / \mathrm{pf}[\mathrm{kNm}], \mathbf{0 . 0 0 0 1}$ [kNm] $)$ | STOP-button $\Delta K<10^{-6 \%} \% / \mathrm{m}$ |

accur1 $=\mathrm{pf}^{\star} 10^{\wedge}$, accur2 $=\mathrm{pf}^{\star} 10^{\wedge 3}$
pf ... precision factor ( $\mathrm{pf}>1$ may increase precision)
$\Delta \mathrm{N}_{\mathrm{a}}=$ Nu2a-Nu1a , $\Delta \mathrm{N}_{\mathrm{c}}=$ Nu1c-Nu2c

Fig.4-69 Convergence and stop criteria of iterations

### 4.3 Joint Characterisation

The sophisticated theoretical background in view of joint representation has already been described in detail in chapter 3 . So the following introduction is primarily intended to practical users of the computer program CoBeJo. Some general repetitions therefore are deliberate.

### 4.3.1 Introduction

### 4.3.1.1 General

Conventionally joints (beam-to-column, beam-to-beam and beam splices) have been treated either as nominally pinned without any strength or stiffness (simple joints) or as fully rigid with full strength (continuous joints); this has been due to incomplete knowledge on joint's moment-rotation behaviour and due to a lack on modelling. In reality both assumptions only represent the limiting cases of the real moment-rotation behaviour of joints. The verification of these assumptions can lead to uneconomical joint detailing as well as to a wrong assessment of the actual structural behaviour in terms of stiffness, resistance and deflections. A modern approach is to develop efficient joint types first and then to take their realistic behaviour into consideration within the frame analysis. An accurate representation of the joint behaviour forms the basis for a correct and safe structural design, where the available joint properties have to be compared with the requirements from the structure at serviceability and ultimate limit states.

Recognition of the importance of integrating the joint behaviour into the global frame analysis in conjunction with modern methods of analysis has resulted in a tremendous number of research activities all over the world aimed at innovation in design. The cooperation and exchange of experience between experts all over Europe in recent years has now enabled a high level of knowledge to be available [36,40,49, 87,94,176].

The following concerns the design of steel and composite beam-to-column joints for continuous and semi-continuous framing. However the concept applies similarly to beam-to-beam joints and beam splices. Moment-resisting joints have to transfer moments and forces between members with an adequate margin of safety. Their behaviour obviously influences the distribution of forces within the structure. Therefore the list of the usual construction elements, namely beams, slabs and columns, has to be extended to include the joints. An overall account of the joint's behaviour would need to recognise its three-dimensional nature. However the presence of stiff continuous floor slabs usually allows neglect of out-of-plane and torsional deformations of the joint. In comparison with the obsolete assumptions of joints being hinges or complete restraints with emphasis on resistance, the modern approach requires the following actions in view of the joints:

Representation (German: Modellbildung):

- Characterisation, determination of the (non-linear) moment-rotation curves; at least the determination of the corresponding key values (initial rotational stiffness, upper moment resistance and rotation capacity)
- Classification, a possible but not obligatory tool for simplification of modelling providing boundary conditions defining the limits for the use of conventional types of joint modelling (nominally rigid or nominally pinned)
- Idealisation, the conversion of non-linear moment-rotation curves into simplified ones
- Modelling, the reproduction of the joint behaviour within the structural analysis.


### 4.3.1.2 Joint characterisation (German: Knotencharakterisierung)

This chapter describes how to derive the M- $\phi$-curves, reflecting the joint behaviour and being the necessary input data for the joint models. The general background for any joint model comprises three separate curves: one for the left hand connection, one for the right hand connection and one for the column web panel in shear. Only later (during the modelling process) the $M-\phi$-characteristic of the shear panel may be shared into a left and right hand side part and then may be assigned to combined springs representing the connection and the shear together.

There are three possibilities of joint characterisation: the experimental one using fullscale joint tests, the numerical one based on finite element calculations and the analytical one, which will be treated now more in detail; this is because it is the only practical approach for every-day design.

## Component method

An analytical description of the behaviour of a joint has to cover all sources of deformabilities, local plastifications, plastic redistribution of forces within the joint itself and local instabilities. Due to the multitude of influencing parameters, a macroscopic inspection of the complex joint by subdividing it into components has proved to be most appropriate. In comparison with the finite element method, these components, which can be modelled by translational springs with non-linear force-deformation response, are exposed to internal forces and not to stresses. The procedure can be expressed in three steps:

- Identification of the contributing components,
- Characterisation of the components (determination of their structural properties)
- Assembly of the component properties or curves to those of the joint.


## Component identification

According to EC3 [60] a (basic) component of a joint is a specific part of a joint that makes - dependent on the type of loading - an identified contribution to one or more of its structural properties. When identifying the contributing components within a joint one can distinguish between components loaded in tension (or bending), compression and shear. Apart from the type of loading one can distinguish between components linked to the connecting elements and those linked to load-introduction into the column web panel (both are included in the "connection") and the component "column web panel in
shear". The nodal subdivision for the most general case of a composite beam-tocolumn joint leads to a sophisticated component model (spring model), see Fig.4-70. There, the interplay of the several components is modelled in a very realistic way. Any composite joints (beam-to-column, beam-to-beam or beam splice), even steel joints can be derived as being only special cases of this general model. The sophisticated component model leads to a complex interplay of components and therefore to iterations within the joint characterisation itself. The solution of these iterations is the objective of the computer program herein.

To avoid such iterations in the codes $[49,60]$ simplifications concerning the component interplay have been adopted resulting in a simplified component model (spring model), see Fig.4-70.


Fig.4-70 Sophisticated and simplified component models (spring models)
Through familiarity with the component model, which represents the interplay of all deformation influences, the principles of economic joint construction with respect to stiffness, resistance, failure modes and ductility can be logically derived. Assuming a specific component with a given resistance and deformation capacity, it is self-evident that by increasing its lever arm the joint's moment resistance increases whilst the rotation capacity decreases. Providing resistance in the compression region far beyond the resistance of the tension region is uneconomic. Similarly strengthening the connecting elements will not make the joint capable of sustaining more load, if failure is already dominated by the column web in tension, compression or shear. It also can be recognised immediately that it makes no sense to combine a relatively weak and ductile slab reinforcement with a stiff and brittle steelwork connection. These examples give ideas on how to use the component model directly for a qualitative assessment of the joint's response. Generally, the most efficient joint is that, for which, due to plastic redistribution within the joints itself, all components are used up to their plastic resistance and which provides a further yield plateau for redistribution of moments within the whole frame system.

## Component characterisation

The component model is based on known force-deformation curves of the individual contributing components, which have to be derived within the component characterisation (derivation of the components' key values or even the full curves). Again this can be done at different levels of accuracy using different tools, such as
component tests (experimental approach), finite element simulations (numerical approach) or analytical mechanical models.

In order to describe the complex component response by easily understandable mechanical models the components given in Fig.4-70 are subdivided into basic joint components leading to the refined component model shown in Fig.4-71. So for example the load-introduction into an encased column web panel is split into the contribution due to the steel section and that due to the concrete encasement.


Fig.4-71 Refined component model

Sophisticated mechanical models describing the basic components' response have been developed at several research centres and validated against component test results and numerical simulations. With the help of comprehensive parameter studies the sophisticated and relatively complex formulae describing the stiffness, resistance and deformation capacity of each basic component can be reduced to easy-to-handle formats. Such simplified formulae are integrated into the codes $[49,60]$.

## Component assembly

The transfer from the force-deformation curves of the individual basic joint components to the moment-rotation curves representing the connection or the shear panel has to be done based on the component model fulfilling the requirements of compatibility and equilibrium. Doing so it is assured that the joint model behaves exactly in the same way than the complex component model with respect to applied moments. Depending on the intended level of accuracy the assembly can be done for the main rotational key values only (initial rotational stiffness, upper moment resistance, rotation capacity) or for the full shape of the resulting $\mathrm{M}-\phi$ curves.

As already mentioned the assembly of the sophisticated component model leads to iteration loops due to the complex interplay of components. These iterations are solved by the "joint characterisation" module of the computer program CoBeJo.

For simplification iterations can be avoided by using the simplified component model used in the Eurocodes, where the sum of all basic component springs can be derived by adding them step by step acting parallel or in series. However also in that case the "joint characterisation" module of the CoBeJo package proofs to be a valuable tool to assemble the overall non-linear spring curves (and not only the initial elastic lines) step by step parallel or in series.

### 4.3.1.3 Joint classification (German: Knotenklassifikation)

After the joint response in terms of M- $\phi$-curves has been derived with the help of the joint characterisation the engineer working in practice has now the possibility either to reproduce the so-gained joint behaviour in the global frame analysis using an advanced joint model or, simplifying, to determine whether already a conventional joint model (nominally pinned or nominally rigid) is able to reproduce the frame response with sufficient accuracy. Such a decision can be made with the help of a joint classification.

In view of the drastic increase of frame software capability the use of advanced joint models (as described in 4.3.1.5) will become more and more natural and therefore the need for a classification tool to fall back on conventional joint modelling will have less and less importance. Therefore the author proposes to get familiar now with the new concept of joint modelling reproducing the realistic joint M - $\phi$-behaviour with (non-linear) springs and to refrain from the old-fashioned approach to regard joints either as nominally pinned or nominally rigid. Consequently, the joint classification is no more of interest.

When still aiming at conventional rigid or hinged joint models the joint characteristics in view of stiffness, strength and rotation capacity have to be compared to classification boundaries [36,49,60]. Depending on the method of global analysis either all or only one of the classification checks becomes decisive [49,60](Table J.2). So e.g. when using elastic-plastic global analysis a simple joint model (hinged) may be used in the case that the joint provides sufficient rotation capacity. In contrast to what is given in the code [60] its stiffness and strength are of minor importance in view of a safe design. A continuous joint model in elastic-plastic global analysis may be used if the joint can be classified to be quasi-rigid and to have full strength. Regardless of that the main problem is the definition of such classification boundaries for stiffness, strength and
ductility based on accuracy criteria for ULS (ultimate limit state) or SLS (serviceability limit state), which should generally be valid for any kind of frame configuration.

### 4.3.1.4 Curve idealisation (German: Kurvenidealisierung)

Depending on the available software either the full non-linear shape of all jointcurves discussed until now or multi-linear simplifications of them can be assigned to the respective joint springs of the joint model. Fig.4-72 shows curve idealisations proposed in EC3 [60]. It is evident that the required input as well as the capability required from the software both increase if high accuracy is intended.

non-linear

tri-linear

bi-linear

Fig.4-72 Possibilities of curve idealisation
The actual moment-rotation response of joints usually is described by means of a non-linear curve. However, the use of such non-linear curves requires sophisticated frame analysis programs. In order to enable a (more simple) linear calculation, e.g. an elastic global frame analysis (this is still the current practice in most European countries), the non-linear curve may be simplified by straight lines. As a conservative assumption each curve lying below the non-linear curve in general may be used in the frame analysis. For example a tri-linear curve is shown in Fig.4-72. The most simple curve is a bi-linear one. In order to use a bi-linear curve which provides the most efficient solutions, comparative studies were performed to calibrate an idealised joint stiffness $\mathrm{Sj}^{*}$. The idealised joint stiffness $\mathrm{Sj}^{*}$ is constant for all values of applied moments smaller than the design moment resistance of the joint. The revised Annex J of EC3 gives guidelines on how to derive such a simplified bi-linear curve: The idealised joint stiffness $\mathrm{Sj}^{*}$ can be calculated by dividing the initial joint stiffness $\mathrm{S}_{\mathrm{j}, \mathrm{ini}}$ with a stiffness modification factor $\eta$ : $S_{j}^{*}=S_{j, \text { ini }} / \eta$.

The stiffness modification factor $\eta$ depends on the type of joint (welded, bolted endplate, bolted flange cleats, contact plate composite joint) and on the joint configuration (beam-to-column or other) and varies between 1.5, 2 and 3.5. Of course, if the applied moment does not exceed the values of $M_{j, \text { elastic }}\left(2 / 3 M_{j, R d}\right)$, a reduction of the initial stiffness $\mathrm{S}_{\mathrm{j}, \text { ini }}$ is not required.

### 4.3.1.5 Joint modelling (German: Knotenmodellierung)

Linked to the joint characteristics there are three types of joint modelling which are closely linked to the method of global analysis and to the classification of the joint: A continuous joint ensures full rotational continuity, a semi-continuous joint ensures only partial rotational continuity and a simple joint prevents any rotational continuity between the connected members (Fig.4-73).


Fig.4-73 Joint modelling - conventional and advanced
It is true that the conventional simplifications of simple or continuous framing result in the advantage of a simple calculation. However as reality lies in between these extreme boundaries, in the age of rapid development of hardware and software it is appropriate to replace the conventional calculation methods by more realistic ones with the joints as additional members with finite dimensions. Depending on the required accuracy several possibilities of joint modelling for global structural analysis can be applied.

## Joint modelling reflecting the actual behaviour

Within a joint two separate sources of deformation have to be distinguished - those due to the connection (load-introduction into the column through the connecting elements) and those due to the shear panel (column web panel in shear due to moment imbalance between the left and right hand sides). To reflect actual joint behaviour a cruciform joint configuration has to be modelled by two separate $\mathrm{M}_{\mathrm{L}}-\phi_{\mathrm{L}}$-curves representing the left and the right connection and one additional $\mathrm{M}_{\mathrm{S}}-\phi_{\mathrm{S}}$-curve (which can be derived from a $V-\gamma$ relationship) for the shear panel, see Fig.4-74.


Fig.4-74 Joint modelling reflecting the actual behaviour
To reflect actual joint behaviour a single-sided joint has to be modelled by two separate M - $\phi$-curves, one for the connection and one for the shear. Fig.4-75 illustrates
equally good possibilities of joint modelling reflecting the actual behaviour by so-called finite joint models.


Fig.4-75 Application of joint modelling reflecting actual behaviour by finite joint models

## Simplified joint modelling

For sake of simplicity a simplified joint model has been introduced in EC3 [60] for every-day design, where the nodal zone is no longer taken into account with finite dimensions. The aim is to represent the overall joint behaviour in one single rotational spring for each joint. Therefore simplifications are introduced as described below.


Fig.4-76 Modelling the web panel and the connections as combined rotational springs
The load-deformation ( $\mathrm{V}-\gamma$ ) curve of the column web panel in shear has to be converted into a moment-rotation ( $\mathrm{M}_{\mathrm{s}}-\phi_{\mathrm{s}}$ ) curve, using the lever arm z. However, the actual shear force $\mathrm{V}_{\text {wp }}$ in the column web panel is not equal to the "local" tension and compression forces within the connection ( $F_{c}=F_{t}=M / z$, see Fig.4-76 and Fig.3-82), due to the "global" shear forces acting in the column. For the simplified modelling according to EC3 this effect is neglected. In case of a single-sided beam-to-column joint configuration, both the curve representing the web panel in shear and that for the connection are added in series to one combined $M$ - $\phi$-curve for the joint, see Fig.4-77. In case of a double-sided joint configuration, two separate but interacting joints may be
modelled. Each of these joints takes into account the behaviour of the column web panel as well as the behaviour of the relevant connection, see Fig.4-76. Thus, the web deformation would be taken into account twice but a so-called transformation factor $\beta$ can be introduced to consider the actual loading of the shear panel in relation to the loading of the individual joints. The validity and accuracy of this procedure is demonstrated in [89].


Fig.4-77 Derivation of a combined rotational spring for the simplified modelling


Fig.4-78 Application of simplified joint modelling by a concentrated joint model
The second difference between the models reflecting actual behaviour and the simplified models concerns the location of the rotational springs. In finite joint models the connection springs are arranged at the edges of the finite joint area, as shown in Fig.4-75. In the simplified joint model the combined joint springs for shear and connections are located at the beam-to-column axes intersection point, see Fig.4-76. It is obvious that the acting bending moment usually increases from the joint edge to the axis intersection point. The consideration of the larger value for the applied moment therefore leads to a more conservative design. On the other hand the flexibility of the extended beam and column stubs within the finite joint area adds an extra rotation to the joints leading to an overestimation of the global frame deformation. This simplification is the same as that, made in the conventional design where simple or continuous modelling is used. The influence of both effects becomes more important with increasing joint dimensions in comparison to the beam span and column height.

These effects can be compensated by a so-called "spring transformation" described in detail in chapter 3.9, which leads to additional refining terms within the formulae characterising the joint properties.

## Subsituting stub method

If the software for frame analysis does not support rotational springs as used for both finite and concentrated joint models they may be substituted by equivalent beam stubs with appropriate bending stiffness. The derivation of the equivalent stub stiffness is shown in Fig.4-79 assuming a constant bending moment along the stub. Thus less deviation to the actual behaviour is achieved if the stub is short.


Fig.4-79 Equivalent stub instead of rotational spring

### 4.3.1.6 Summary

So far joints have been simplified by treating them either as fully continuous or as pinned in structural analysis. Joint detailing had to be adapted to these rough calculation assumptions, which could be expensive. Nowadays there is the opportunity to follow the opposite but much more efficient way: Economic joint types are developed first to ease construction and then their realistic behaviour is determined and taken into account within the global frame analysis. Thus, due to knowledge on joint representation and powerful software the working expenditure shifts from the construction to the cheaper calculation process. It is shown in Fig.4-80, how the moment-rotation behaviour of joints can be represented in structural analysis in a practical way.

It has been described briefly how to derive the joint's response ( $\mathrm{M}-\phi$-curves), which forms the necessary input data for the frame calculation, focusing on the analytical approach - the so-called component method. Both, sophisticated models as well as the simplifications used in Annex $J$ to Eurocode 3 and the intended corresponding Annex to Eurocode 4 have been discussed. However any new design method which involves even minor changes to existing practice will only be adopted if genuine and significant benefits can be demonstrated, justifying the increase of design efforts. Such benefits from semi-continuous frame design can be: A reduction of beam dimension in comparison to hinged girders and therefore a reduction of cladding costs and easier integration of services; and in comparison with continuous construction a remarkable reduction of construction and erection costs. Naturally these benefits gain importance
for unbraced frames. Furthermore it is important to be conscious that the acceptance of this new philosophy will be improved by providing design guidelines and every-day design aids like manuals, tables and software.


## GLOBAL STRUCTURAL ANALYSIS

Fig.4-80 Practical procedure of joint representation in structural analysis

### 4.3.2 General

The module "Joint characterisation" of the CoBeJo-program package is a design aid, providing the joint curves on the basis of the analytical component method approach (Fig.4-81). It is assumed that the force-deformation behaviour of all contributing components in terms of F -w-curves is already known (from mechanical models, component tests, FE-calculations). The computer program iteratively assembles all these component curves fulfilling equilibrium and compatibility.

The reasons not to rely on an existing FEM program but to develop such a selfstanding joint module have been:

- the high demands on pre- and post-processing especially adjusted to the joint characterisation problem
- the independence of such little disseminated and expensive programs
- the plan to integrate this iterative assembly procedure into the Module Bank Program [118] where the component characterisation is already done automatically based on mechanical models
This joint characterisation module should serve as a tool both for scientists and advanced practitioners to "play" with different joint configurations and so to improve acceptance of the new joint representation philosophy. Therefore special attention has been directed to a clear graphical display of the gained results and also to the coherence between individual component characteristics and the joint response. Differently to FEM-programs the solution of this non-linear spring assembly problem in the CoBeJo-Joint-module is solved iteratively with an energy criterion step by step considering the load history and not using a step by step modified stiffness matrix.

The main task of the CoBeJo-Joint-module is the full assembly of non-linear translational component springs to an overall joint response including post limit effects of the contributing components as well as failure of individual components. In the future this tool should give the following possibilities to researchers:

- Comparison between different assembly methods due to different component models (see Fig.4-70)
- Comparison between the full spring assembly and the assembly in view of the key values stiffness and strength only
- Check of curve idealisation methods
- Influence of overstrength effects of single components on the overall joint behaviour (especially in view of rotation capacity for different failure modes)
- Study of failure modes
- Investigations in view of available rotation capacity

Within the scope of this work the assembly has only been realised for symmetrically loaded joints ( $\beta=0=1-\mathrm{M}_{\text {left }} / \mathrm{M}_{\text {right }}$ ). Later on also fully unbalanced joints $(\beta=1)$ should be covered. Further it is assumed that the joint configuration in view of contributing components and their behaviour is symmetrical and that the joint is subjected to hogging bending only.
The general structure of the module can be taken from Fig.4-82.
based on the Component Method

## Joint Configuration



Component Identification


## Component Characterisation


e.g. based on analytical mechanical models

Component Assembly

iteratively fulfilling equilibrium and compatibility


CoBeJo Joint

Fig.4-81 Joint characterisation using CoBeJo-Joint

## Module "Joint Characterisation"



Fig.4-82 Structure of the module "Joint Characterisation"

### 4.3.3 Joint management



Fig.4-83 Start-up window of the module "Joint Characterisation" (start.frm)
Fig.4-83 shows the start-up window of the module "Joint Characterisation". It serves for the file-management of joint data and gives the link to the windows for the joint input and the joint calculation (assembly for $\beta=0$ : balanced joint loading). A special joint configuration (see Fig.4-84) consisting of one steelwork row in tension and one layer of slab reinforcement in tension has been chosen for the following examples.


Fig.4-84 Example joint configuration

An image box in the centre of the start-up window shows the actual joint configuration as a symbolic sketch (Fig.4-85). The white area marks the joint area. Stiff separation bars are drawn at the boundaries to the adjacent beams (sb1) and in view of the steelwork rows between the connecting elements and the elements subjected to load introduction into the web panel (sb2). The bars sb2 are located directly at the joint edges (white area), the bars sb1 and therefore also the area of the connecting elements for illustrative reasons fictitiously is extended into the beam area (in reality the distance between the bars sb1 and sb2 is zero). With the scroll bar "connection size" this illustrative ratio between the fictitious size of the connecting elements and the joint width can be adjusted between $1 / 60$ and 1/2. An analogous illustration method has been chosen for the web panel in shear. Strictly the components subjected to load introduction into the web panel and those of the web panel in shear (in case of moment imbalance) are sharing the same area within the joint. To avoid any misleading overlap of component springs the joint width (white area) fictitiously is shared between the load introduction and the shear components. Again the illustration ratio between the shear panel size and the joint width can be adjusted between $2 / 10$ and $2 / 1.1$ with the scroll bar "shear panel size". As described in detail in chapter 3.4 the following components may be identified within a joint configuration:

## Component

basic components
acc. to Fig.4-71
1,2,3,4
5
6,7
8,9,10
11
13a-g
12
$14,15,16$

Sys Flexibility of the adjacent columns or the frame system

Each of these components symbolically is drawn as a horizontal bar at the height of its relevant lever arm. The drawing width of the bars represents the stiffness ratio between the several components and the colour of the bars represents their strength: light blue means relatively low strength, black means relatively high strength. (The system spring Sys is not considered for these illustrative ratios.) This should ease a first check of the given joint data. The described joint module of CoBeJo enables the input of up to 5 rows of steelwork connections (springs C,D) and 2 layers of reinforcement (springs E,F,G). Finally the coupling height between the joint as a whole and any adjacent beam member has to be defined for a semi-continuous global beam analysis with beam and joint elements placed one behind the other (this module will follow in the future).


Fig.4-85 Joint configuration sketch

All joint data including joint name, joint-ID, joint width, height of coupling point and all identified components with their force-deformation behaviour and relevant lever arms are saved to a file with the default-extension "*.jip". The input file of the example joint can be seen on page 263. Clicking on the "Load joint" or "Save joint" buttons activates the window for the file management (Fig.4-86). Apart from the possibility to load or save joint data buttons are provided for a small variety of disk manipulations like "Create Directory", "Remove Directory", "Rename File", "Copy File" and "Delete File".


Fig.4-86 File management (file.frm)

Before saving, before calculating and after loading a joint configuration an automatic joint check will be done with a final message box summarising all noticed errors and warnings. In the following an enumeration of the checked data is given:
error checks (in case of errors no calculation will be allowed):

- complete compression row (springs $\mathrm{A}+\mathrm{B}$ )?
- at least one complete tension row (springs $\mathrm{C}+\mathrm{D}$ or $\mathrm{E}+\mathrm{F}$ )?
- existing incomplete tension row?
- slip spring given for both reinforcement rows or for none?
- defined shear spring?
warning checks (in case of warnings a further calculation is possible):
- slip spring defined for reinforcement row (otherwise slip value has to be given)?
- lever arm of reinforcement rows higher than that of steelwork connections?
- comparison between joint height and effective lever arm of shear panel?
- defined system spring (otherwise it will automatically be set infinitely stiff)?

It was generally tried to keep the program self-explaining, anyway also the other possible operations within the start-up window at least should be mentioned here. A special text field is reserved for a joint name and the button "New joint" deletes the actual joint input. While moving the mouse pointer over the joint sketch the actual position (width, height) is indicated.

Finally it should not be forgotten to mention the precision factor, which as default value is set equal to one. Generally all accuracy pre-settings within the iteration loops have been defined as a compromise between precision and calculation time. For each iteration result at the end the final error will be declared. So if this indicated error should exceed the desired limits the accuracy might be improved by raising this precision factor.

The button "Joint input" or "Calculations" will close the start-up window and open that for the joint input or the joint calculation, which will be described in detail in the following chapters.

## Input file: thesis.jip

| Joint data |  |
| :--- | :--- |
| Thesis |  |
| Joint-ID | 990319183637 |
| Check-indicator | 1 |
| joint width | 100 |
| Coupling point $y=$ | 0 |

dimensions: $k N, m m$
keyword
joint name
identification for coherence between input and result files
indicates successful joint check
$b_{j}$
height of coupling point to beam elements

COMPRESSION REGION

| Spring $\boldsymbol{A}$ | A cwc |
| :---: | :---: |
| 0 | 0 |
| 0.1 | 100 |
| 1 | 300 |
| 3 | 300 |
| -1 | 0 |
| -1 | 0 |
| -1 | 0 |
| 1 | 0 |
| unloading | 1000 |
| Spring B | B bfc |
| 0 | 0 |
| 0 | 200 |
| 1 | 0 |
| -1 | 0 |
| -1 | 0 |
| -1 | 0 |
| -1 | 0 |
| -1 |  |
| 1 |  |

TENSION REGION
Steelwork rows -------------
Row no. 1 ------- $y=\quad 100$
Spring C1 C cwt
0 C cw
0
85
0.1

130 130

1 Springtype, Name, existing (=1) w-F-relationship

Continuation of input file:


### 4.3.4 Joint input



Fig.4-87 Window for the input of the joint configuration data (jnput.frm)
The window shown in Fig.4-87 enables the input of the joint configuration data. The following self-explaining options are available: definition of a joint name by a text box, with "New joint" all actual data can be deleted to start with a new joint configuration and the "Quit"-button closes the input window and leads back to the start-up window.

The "info"-button provides the user with some general information about the meaning of the component springs A-H and Sys:
A Web panel in compression
B Beam flange in compression (infinitely stiff, limited resistance, see Fig.4-88)
C Web panel in tension
D Connecting elements in tension (beam flange and endplate in bending, bolts in tension,...)

E Reinforcement within the full column depth in tension (pay attention: the spring E spans over the whole joint width $b_{j}$ and not only over $b_{j} / 2$ )
F Redirection of unbalanced forces (only acting in compression; therefore only on less heavily loaded side). The spring characteristic itself is independent of the load ratio left/right (see Fig.3-12: spring 13=F for fully unbalanced situation $\mu=1 \approx \beta$ )
G Slip between steel beam and slab at column front.
For an isolated joint characterisation an assumption has to be made for the w-F characteristic of this slip spring representing the weakness of the horizontal shear joint between the steel beam and the concrete slab outside of the joint area itself. $F$ is equal to the internal shear force $N_{i}$ (see 4.2.5) and $w$ is the corresponding slip value, both at the beam end (=joint edge). So this w-F characteristic of spring $G$ would have to be derived by a sophisticated or simplified beam calculation considering incomplete shear interaction (see 1.4.5 and Fig.3-13). Strictly there is an interaction between the beam and the joint response and therefore only an overall semi-continuous beam-including-joint analysis would give really realistic results. Such a tool will be provided by the third CoBeJo module "Global Analysis" (see Fig.4-1) combining beam and joint elements. There the slip will be derived as the difference of the displacements between beam and slab along the whole beam span. This slip value then will be taken for the corresponding joint characterisation ignoring any defined w-Frelationship. So only for such an overall analysis no slip response $G$ has to be entered.
H Shear influence. Strictly depending on the actual lever arm z, which here is assumed to be constant during joint moment increase. The correct lever arm could be derived from

$$
z=\frac{M}{F_{\text {compression }}}=\frac{M}{\sum F_{\text {tension }}}
$$

Sys Flexibility of the adjacent columns. For a bare joint analysis this system spring has to be set infinitely stiff.
For really detailed information on the individual components it is referred to chapter 3.4 herein.


Fig.4-88
Response of compression spring $B$

Now the definition of the joint input data will be described: Both for joint sketches true to scale and for a final integration of joint elements into semi-continuous beam
calculations (planned for the future) the joint width $b_{j}[\mathrm{~mm}]$ has to be entered. For the later assembly of joint and beam elements the vertical distance between the coupling point and the compression row has to be given as "element coupling point" [mm].

Then the data of each identified component A-H has to be defined, including the lever arm, the force-deformation response and the unloading stiffness. (For further calculations in the case of balanced loading ( $\beta=0$ ) the components $F, H$ and Sys have no influence on the results as they would only contribute in the case of moment imbalance. Despite at least some fictitious data have to be entered to enable a successful joint check.) Pay attention: if the overall M- $\phi$ behaviour should be studied at the design level already the F-w curves of the contributing components should be given on the design level including safety-factors. Reversely giving characteristic component curves results in characteristic joint curves and giving actual component curves (e.g. from test results) leads to actual joint curves.

Fig.4-89 shows the corresponding tools for this component input within the joint input window.


Fig.4-89 Component spring data

Except for the compression row (components A and B ) and the shear spring H the lever arm of the actual component has to be defined being the vertical distance to the centre of compression (positive value). Then a name may be defined for the actual component, followed by the input of the force(F[kN])-deformation(w[mm]) response by the help of a grid box with up to ten rows (so the non-linear shape of a component's $\mathbf{w}$ -F-curve is restricted to a polygon with at maximum ten nodes). (By way of exception for the compression spring $B$ not a polygon but only an upper strength value has to be entered; the response of spring B will then be set infinitely rigid with a ductile plastic plateau at the given strength limit; see Fig.4-88.)

Within a spring data check it will be inspected if the following principles are fulfilled:

- the w-F-curve must start with $\mathrm{w}=0 / \mathrm{F}=0$
- at least one further w/F-point has to be given
- the w-values must be increasing from one row to the next (monotonous curve)
- the F-values must be positive

With the button "Save" the actual w-F-curve can be stored to a spring file (*.spr) in the format shown in Fig.4-90. So the w-F-curve of the actual component may either be entered directly value by value or such a saved spring curve can be loaded. The "Load" and "Save" buttons will activate a file management window quite similar to that for the handling of joint files shown in Fig.4-86. Further there is the possibility to use the MS Windows Clipboard for a data exchange to other applications (e.g. MS Excel). For that purpose the buttons "Copy" and "Paste" have been provided. Concerning both the load and paste command it will be checked automatically if the new curve starts with $\mathrm{w}=0 / \mathrm{F}=0$ and in the case of very long numbers eventually some digits behind the comma may be cut. With "Clear" the whole w-F-curve can be deleted. Concerning the manual input of values within the w-F-grid the following actions are possible: the desired grid element may be chosen using the cursor buttons or the mouse, then the value has to be defined using the number keys and the dot "." as decimal separator. Enter leads you to the next grid box. With the delete button on your keyboard all marked grid elements will be reset. A single mouse click on a grid element activates the overwrite mode, whereas a double click enables to modify an already entered grid value, e.g. using the backspace key.
Finally the unloading stiffness $[\mathrm{kN} / \mathrm{mm}]$ has to be defined, which normally is equal to the initial (elastic) stiffness of the curve (this can be done automatically using the button beside the input box).
To ease checking of the actual input values a true to scale sketch of the w-F-curve will be drawn below the input boxes. An automatic refresh is performed after each input modification. The unloading stiffness is visualised by a light blue line through the origin.

Spring file: c.spr

| C cwt |  | Spring name <br> 0 |
| :---: | :---: | :---: |
| 0.1 | 0 | $w$-values $[\mathrm{mm}], F$-values $[\mathrm{kN}]$ |
| 0.6 | 130 |  |
| 1 | 130 |  |

Fig.4-90 Spring file *.spr

After all data of a contributing component has been entered this information just has to be assigned to such a component by a single mouse click on the relevant area within the component model sketch (Fig.4-91). For the tension springs C,D,E,F,G additionally also a row number has to be given. A hook and a different mouse pointer then are indicating a successfully defined component. As any joint is assumed to have a symmetrical component configuration both components (left and right side) are defined
with only one click on one of both sides. By assigning an empty w-F-box to a component this component will be removed.


Fig.4-91 Component assignment by a single mouse click

As the w-F-values of the shear spring $H$ have to be converted into $\phi$ - M -values an estimated lever arm $z$ has to be entered ( $\phi=w / z, M=F^{\star} z$ ). Strictly the lever arm of the shear panel changes during load increase and therefore also the shear response strictly would have to be adopted to that, however this normally minor influence will be neglected.

As mentioned before with a mouse click the data within the input boxes (lever arm, spring name, w-F grid and unloading stiffness) will be assigned to one of the components. Pressing "SHIFT" while clicking on a component (Fig.4-91) causes the reverse: the stored data of the relevant component is copied to the data boxes. This can be used for component checking, copying or for modifications of the component response or the lever arms. Pay attention to the fact that the component actually only will be modified after the modified data has again been assigned to it with a single mouse click on the same component (without shift button). Completed tension rows can be copied with the "Copy row" button. "Automatic unload" adjusts the unloading stiffness of all actually stored components to their initial stiffness.

The system spring Sys has a special meaning not directly related to the joint behaviour itself. Therefore it is only of interest for a semi-continuous beam calculation using an assembly of beam and joint elements. This spring takes into consideration all flexibilities coming from the columns or even the frame system adjacent to the joint. So for a bare joint calculation this stiffness has to be set infinite (will be done automatically if no $\phi$ - M -values are defined). The $\phi[\mathrm{mrad}]-\mathrm{M}[\mathrm{kNm}]$ response either can be entered directly using the input grid (also M values must be monotonous) or use can be made of a special calculation window shown in Fig.4-92. Three types of column support conditions are available. By giving the necessary values $h_{a}, h_{b}, E I_{a}$ and $E I_{b}$ the corresponding system stiffness $S$ will automatically be calculated. "Accept"ing this stiffness will add a $\phi-\mathrm{M}$ row to the input grid of Fig.4-89 ( $\phi=1 \mathrm{rad}, \mathrm{M}=\mathrm{S}^{*} \phi$ ) and assign this
data automatically to the system component in Fig.4-91. If $S=0$ the stiffness will be set infinite.


Fig.4-92 Calculation window for the system stiffness (syscase.frm)

It should not be forgotten to mention that by saving a joint in the start-up window (Fig.4-83) it will get a unique joint-ID (identification number), representing the exact saving time. This enables a clear assignment between joint data files and result files later on. For that reason the actual joint-ID gained by saving or loading a joint configuration will be lost again if the joint data is already slightly modified with the help of the input window.

Finally there remains the button "Linear grouping" which closes the input window and leads to the window for linear grouping of translational springs.

### 4.3.5 Linear Grouping



Fig.4-93 Window for the linear grouping (=assembly of translational springs) (transgrp.frm)
With the help of the linear grouping window (Fig.4-93) translational springs directly acting parallel or serial can be assembled. This can be used either as a completely separate tool apart from any joint investigations to add any curves (not only w-F curves) vertically or horizontally or to evaluate average values of two curves (as will be described later) or it is used directly as a kind of pre-assembly (pre-calculation) in view of the joint input. Remember e.g. that the component $A$ is set together by the basic components $1,2,3,4$ in direct parallel or serial interplay (acc. to Fig.4-71). The linear grouping tool is also available as a self-standing program.

First of all the principles of parallel and serial spring assembly should be given (see also chapter 3.7.1 herein). The definition of parallel spring interplay is the equal deformation of both springs $\left(\mathrm{w}=\mathrm{w}_{1}=\mathrm{w}_{2}\right)$, the corresponding forces of the two springs ( $\mathrm{F}_{1}$, $F_{2}$ ) have to be added to gain the overall force. So the w-values are regarded as common "basic values". In graphical sense the two w-F-curves are added in vertical direction. Therefore the stiffness and resistance of the parallel spring group exceeds that of both individual springs and is mainly influenced by the stronger one. When reaching the failure of one spring the bearing capacity of the parallel group drops to the resistance of the other spring and therefore the minimum deformation value of the
participating components has to be taken for the deformation capacity of the group. The definition of serial spring interplay is the equal force of both springs ( $F=F_{1}=F_{2}$ ), the corresponding deformations of the two springs ( $\mathrm{w}_{1}, \mathrm{w}_{2}$ ) have to be added. So the Fvalues are regarded as common "basic values". In graphical sense the two w-F-curves are added in horizontal direction. Therefore the stiffness of the group is even smaller than the minimum of both components, the resistance is that of the weaker spring but as a positive effect - the deformation capacity increases (Fig.4-94).
parallel springs

serial springs


minimum of deformation capacity


minimum of resistance decrease of stiffness
increase of deformation capacity
$F=\min \left(F_{1}, F_{2}\right)$
$1 / C=1 / C_{1}+1 / C_{2}$ $\mathrm{w}_{\mathrm{u}}=\mathrm{w}_{1}+\mathrm{w}_{2}$

Fig.4-94 Assembly of translational spring groups (linear grouping)

It is obvious that spring curves can only be added within a common area of basic values (within a common w-range for parallel grouping or within a common F-range for serial grouping). Furthermore special attention has to be paid to curves with a falling branch (increase of basic values followed by a drastic decrease = post-limit effect) as shown in Fig.4-95 for the case of a serial spring sequence. In that case the assembly has to be done in two steps: The decisive curve of the weaker spring will be split into two parts - the increasing and the decreasing one. Then first the increasing parts of both curves have to be added, afterwards the decreasing part of the weaker spring will be added to the decreasing (unloading) line of the stronger spring. Normally the unloading stiffness is equal to the initial stiffness.


Fig.4-95 Serial spring assembly with decreasing branch

After these general considerations the handling of the "transgrp"-module will now be described. First of all two spring curves ( $w-F$ ) have to be defined using the input boxes shown in Fig.4-96. A textbox is reserved to a spring name. A grid of at maximum 1000 rows allows the input of the w-F-relationship. The large row number has been chosen in view of an evaluation of test curves. The handling of the grid element is the same as already described in 4.3.4 (Fig.4-89) with the possibilities of a manual input, loading and saving of curves and the use of the MS windows clipboard with the copy and paste buttons. As the basic curve values ( w for a parallel assembly and F for a serial assembly) have to increase monotonously (at least within the increasing branch of the curve) for a correct calculation, it might be of help (especially in view of test curves) to re-sort them by w or by F using the buttons "sort w" or "sort F". A true to scale graphical check of the entered curves can be done with the button "Draw". The first spring curve is drawn in blue colour, the second one is drawn in green.


Fig.4-96 Spring curves - input data and graph
Then the user has to decide which calculation method should be used for the given curves. There are the following possibilities (Fig.4-97):

- spring no. 2 can be added to ("+") or subtracted from ("-") spring no. 1
- the assembly can be done parallel or serial
- the basic values can be taken step by step from both curves (to consider all bends of the resulting curve) or only those of spring no. 1 or spring no. 2 can be considered; this second possibility has been provided in view of the evaluation of test curves with a plenty of curve points and therefore already only a small increase of the basic values from row to row.
- finally there is the possibility to multiply the resulting curve values with a w- and Fmultiplier; normally both of these will have to be set equal to one, however a multiplier of 0.5 enables to determine average curves as shown in Fig.4-98.


Fig.4-97 Determination of calculation method


Fig.4-98 Average curves using the linear grouping module

Now the calculation itself can be started with the button "Calculation". First the existence of a common basic data field (common w-range for parallel assembly or common F-range for serial assembly) will be checked, followed by a check if the basic values (w for parallel assembly and F for serial assembly) are increasing monotonously. If a value decrease is observed there are two possibilities: If the decrease is bigger than one percent of the maximum range (drastic decrease=post-limit effect) the curve is split into an increasing and a decreasing branch as shown in Fig.4-95. Then the unloading stiffness of the stronger spring has to be entered (initial stiffness given as default). This unloading branch will then automatically be integrated into the value table of the respective spring and the graph. In the case of a minor decrease this may be tolerated
by the user giving some acceptable monotony-limit (especially for test curves alternatively to a curve re-sort).

After a successful calculation the result panel shown in Fig.4-99 will appear including the spring name, a grid element with at maximum 2000 rows showing the calculated curve values (which cannot be edited any more) and a graph, where the result curve is drawn in red. The values of the resulting curve can be copied to the clipboard with "Copy" or saved to a file with "Save". Moving the mouse over the graph shows the actual mouse position in view of w and F. After clicking on "Zoom in" the corners of a zoom window within the actual graph can be chosen (two single clicks on the diagonal zoom window edge points). The option button "Full data area" reproduced the maximum data field again.


Fig.4-99 Result panel for parallel and serial grouping

Sometimes it might be of interest to approximate a resulting curve with a plenty of bends by a polygon. So a graphic approximation tool has been provided shown in Fig.4-100. The w-F values of such an approximation curve may either be entered into a special grid box or the corresponding values to the actual mouse position within the result graph can be added to the value table with a simple mouse click. After two rows of the approximation curve are entered this curve will automatically be drawn in the graph (starting in yellow for the first ten rows and then continuing in white colour). With a double mouse click or by clicking on "New approx.curve" the whole approximation curve can be deleted. As described for the other spring curves also the approximation curve can be saved, loaded, copied or pasted.


Fig.4-100 Curve approximation by a polygon

Summarising this tool for the assembly of translational springs acting together parallel or serial can even directly be used for the full spring assembly (fulfilling equilibrium and compatibility) when applying the simplified Eurocode component model (Fig.4-101). For that purpose the individual components simply have to be added step by step, taking into consideration their relevant lever arms: First the effective behaviour of the tension rows has to be determined adding C1+D1, D2+D2 and E/2+G in series ( $\mathrm{E} / 2$ means the curve of the overall E -spring, where the w -values are divided by 2 to take into consideration the left and right influence). Then these effective tension rows have to be converted from $w-F$ curves to $\phi-M$ curves bringing in the lever arms: for each of the rows the w -values have to be divided by $\mathrm{z}_{i}$ and the F -values must be multiplied with $z_{i}(\phi=w / z, M=F z)$. Afterwards all these $M-\phi$ curves representing the individual tension rows have to be added parallel leading to an equivalent overall tension behaviour in terms of $M-\phi$. Using the formulae given in [60] or the equivalent counterpart below at least an initial equivalent lever arm $\mathrm{z}_{\mathrm{eq}}$ can be calculated.

$$
z_{\text {eq }}=\frac{M}{\sum F_{\text {tension }}}=\frac{M}{F_{\text {compression }}}
$$

Now it has to be checked if the strength of the compression spring B is high enough to activate all these tension forces (equivalent rotational tension spring and converted rotational compression spring $B$ with lever arm $z_{e q}$ in series). Finally also the translational compression spring A has to be converted into a rotational one with the lever arm $z_{\text {eq }}$ and added in series to the previous result. In case of full moment imbalance $(\beta=1)$ the behaviour of the shear panel H has to be added in series too. As this spring assembly according to the simplified Eurocode component model has not
been of further interest within the scope of this work so far no automated procedure has been provided for that step-by-step assembly.


Fig.4-101 Simplified component model (Eurocodes)

The step-by-step linear grouping method should also be used to determine analytically the full rotational joint response of joint configurations without any steelwork row in tension (so-called boltless joints or contact-plate joints, see Fig.4-102). Such an analytical assembly method has the advantage of perfect accuracy in comparison to the iterative assembly (see chapter 4.3.7) leading to some deviations.

The analytical assembly of the contributing components for such a boltless joint has to be done in the following way: First the w-F-curve of the spring $E$ has to be shared into a left and right hand side part leading to a spring $E / 2\left(w_{E / 2}=W_{\mathrm{E}} / 2, \mathrm{~F}_{\mathrm{E} / 2}=\mathrm{F}\right)$. Then the $w$-F-curves of the springs $E / 2, G, A$ and $B$ have to be added in series step by step (eventually considering decreasing branches). The resulting w-F curve has to be converted into a $\phi-\mathrm{M}$ curve using the following formulae:

$$
\phi=\frac{w}{z} \quad M=F \cdot z
$$



Fig.4-102 Linear grouping step-by-step for boltless joints

An analogous linear step-by-step assembly can be applied to stiffened (by stiffeners or column concrete encasement) joints with only steelwork connections and no slab reinforcement (Fig.4-103): Springs D1 and D2 are added parallel taking into consideration the respective lever arms, then spring $B$ and finally spring $A$ are added serial using the equivalent lever arm $z_{\text {eq }}$.


Fig.4-103 Linear grouping step-by-step for stiffened stee/work joints

Finally the button "Quit" closes the linear grouping window again and leads back to the joint input window (Fig.4-87).

### 4.3.6 Basic joint calculations

Clicking on "Calculations" within the start-up window (Fig.4-83) opens the main calculation window (Fig.4-126). However to explain the mechanical and mathematical background of the joint calculations it seems to be more favourable to start here with the "basic calculations" (button) offered by the window shown in Fig.4-104. Furthermore this proceeding is better in line with the historical development of the CoBeJo-Joint module.


Fig.4-104 Basic joint calculation window (assembly0basical.frm)
When entering the calculation windows the springs $E$ and $G$ will be added in series automatically ( $G+E+G$ ) as a kind of pre-assembly according to the principles given in the chapter of "linear grouping" (4.3.5), if necessary taking into consideration that decreasing branches of one spring have to be added with the respective unloading line of the other spring. If no spring G has been defined a specific slip value gained from a previous beam calculation considering incomplete shear interaction has to be entered. So in this case $w_{G}$ will be equal to this specified slip value independently of the actual force in the reinforcement layer according to a w-F characteristic shown in Fig.4-105.


Fig.4-105 w-F characteristic of missing G spring

### 4.3.6.1 Fundamental calculation ("basic0")



Fig.4-106 Fundamental calculation window

Fig.4-106 shows the fundamental calculation window being a part of Fig.4-104. Three fundamental joint deformation values have to be entered defining a unique (symmetrical) joint deformation state:

- $w_{0}$ is the deformation of the compression row (dent into the column web panel)
- $\Psi$ is the rotation of the column front centre line (loadintroduction, separation bar sb2, see Fig.4-85)
- $\phi$ is the real joint rotation including the rotation of the loadintroduction $\Psi$ and the additional rotation due to the weakness of the connecting elements $\vartheta$ (see Fig.3-42).

For the herein assumption of a balanced loading configuration ( $\beta=0=1-\mathrm{M}_{\text {left }} / \mathrm{M}_{\text {right }}$ ) these joint deformations will appear symmetrically also at the left hand side of the joint. With the "calculation" button the following fundamental calculation will be launched (basic0 in global.bas):

## Individual spring deformations

Knowing the values $w_{0}, \Psi$ and $\phi$ the individual deformations of all identified springs can be calculated out of the compatibility conditions:

$$
\begin{aligned}
& \mathrm{w}_{\mathrm{A}}=\mathrm{w}_{0} \\
& \mathrm{w}_{\mathrm{B}}=0 \\
& \mathrm{w}_{\mathrm{C}_{\mathrm{i}}}=\Psi \cdot \mathrm{y}_{\mathrm{i}}-\mathrm{w}_{0} \\
& \mathrm{w}_{\mathrm{D}_{\mathrm{i}}}=(\phi-\Psi) \cdot \mathrm{y}_{\mathrm{i}} \\
& \mathrm{w}_{\mathrm{E}+2 \mathrm{G}_{\mathrm{k}}}=2 \cdot\left(\phi \cdot \mathrm{y}_{\mathrm{k}}-\mathrm{w}_{0}\right)
\end{aligned}
$$

## Corresponding spring forces

With that information the corresponding forces of the individual translational springs $A, C_{i}, D_{i}, E_{k}, G_{k}$ can be determined from the relevant w-F-curve as shown in Fig.4-107 (using the global routine interpolation). If the deformation is less than zero the force is zero and an indicator flag "invalid force sign" will be set (compression spring in tension or tension spring in compression). If the deformation is bigger than $\mathrm{w}_{\text {max }}$ (the maximum w-value of the spring) the corresponding force again is zero and the indicator flag "spring failure" is set.


Fig.4-107 Interpolation of the corresponding force to a given spring deformation

In view of a progressive load history it is possible that the deformation of a spring decreases again although $\phi$ is increasing. Such a decrease of w causes a linear unloading instead of going back the w-F curve directly. To take into consideration such effects there has been provided the button "pre-defo" to enter recent w-F-points ( $w_{\text {hist }}$, $F_{\text {hist }}$ ) of all contributing springs representing a decisive previous step of a load history (pressing "shift" and "pre-defo" will reset the pre-deformations). So within the interpolation routine it will be checked for each spring if the previous (maximum) deformation value is exceeded by the actual deformation or not. If the actual one (w) is bigger then the previous one ( $\mathrm{w}_{\text {hist }}$ ) the interpolation is performed according to Fig.4-107. Otherwise instead of a curve interpolation the corresponding force will be calculated using the unloading stiffness of the respective spring (see Fig.4-108). If $w_{\max }$ has been exceeded once during the load history the force of this spring from then on will always be set zero, because such a spring rupture cannot be repaired again.


Fig.4-108 Interpolation considering the load history

## Force degrees

To get a feeling about the percentage of use of the identified components at the actual deformation state the corresponding force degrees ( $F / F_{\text {max }}$ ) are calculated.

## Equilibrium conditions

From the condition $\Sigma \mathrm{H}=0$ (horizontal forces) the remaining force in spring B can be calculated with the help of cut 1 (see Fig. $4-109$ ). Further the internal moment $\mathrm{M}_{\text {internal }}$ can be gained with cut 1 . Cut 2 leads to the normal force $\mathrm{N}_{\text {joint }}$ and the moment $\mathrm{M}_{\text {joint }}$.


Fig.4-109 Internal forces from equilibrium conditions

## Presentation of results



Fig.4-110 Resulting window from fundamental calculation
After these calculations have been carried out the results corresponding to the chosen deformation state ( $\mathrm{w}_{0}, \Psi, \phi$ ) are presented in a window like Fig.4-110 comprising the following information:

- The primary results are the actual normal force $\mathrm{N}_{\text {joint }}=\mathrm{N}$ and the bending moment $M_{j o i n t}=M$ at the joint edge as well as the internal bending Moment $M_{\text {internal }}=M_{i}$ presented in Fig.4-111. From the general assumption of upper tension rows and a compression row at the lowest level it follows that $\mathrm{M}_{\text {joint }}$ is always negative (hogging bending).


## CALCULATION RESULTS:

$$
\begin{gathered}
\mathrm{H}=-\mathbf{0 . 0 0 2}[\mathrm{kH}] \\
\mathrm{M}=-18.337[\mathrm{kHm}] \\
\mathrm{Mi}=0.000[\mathrm{kNm}]
\end{gathered}
$$

Fig.4-111 Primary results from the fundamental calculation

- The graphical illustration of the deformed joint is given in Fig.4-112. Additionally to the joint graph already explained in chapter 4.3 .3 (Fig.4-85) also the short names of all contributing components are given as well as the respective "degrees of forces" in the form of small lines in the middle of each component spring. A horizontal line means that there is no force acting in a spring ( $0 \%$ ) and a vertical line indicates that the maximum force in a spring is reached ( $100 \%$ ). A wrong force sign within a
component (compression spring in tension or tension spring in compression) is expressed by a thick horizontal line, a thick vertical line indicates a spring failure. These force degrees should improve the first optical assessment of the joint response. With three option buttons (dark,light,none) the format of these force degrees can be selected. Apart from the possibilities to manipulate the illustration size of the shear panel and that of the connection the deformation factor can be defined between 0 and 50 and the scale of the overall joint graph can be chosen between 0.1 and 1 using the respective scroll bars. Furthermore the rotation centres of the two separation bars sb1 and sb2 (acc. to Fig.4-85) are drawn as red circles. Moving the mouse over the joint graph gives the actual mouse pointer coordinates.


Fig.4-112 Graph of the joint response

- A third area within the result window (Fig.4-110) is dedicated to the individual spring results (see Fig.4-113). Clicking on the table which lists all identified components shows their actual deformation $w$, the force $F$, the degree of force $F / F_{\max }$ and a graph of the w-F curve including the actual point as a red circle. Following the principles given in Fig.4-107 and Fig.4-108 the actual point must not always lie on the curve itself. With the option buttons "same scale" or "max scale" it can be chosen if the w-F curves of all components should be drawn in the same scale for a better direct comparison or if for each spring curve the maximum possible scale should be used.


F

same
scale $C \quad \begin{aligned} & \text { max } \\ & \text { scale }\end{aligned}$
Fig.4-113 Individual spring response
With "Done" in Fig.4-110 the result presentation can be finished to start a new calculation.

### 4.3.6.2 Fundamental graphs



Fig.4-114 Fundamental graph and iteration window
Clicking on the button "Graph" produces a quasi three dimensional diagram (vertical layers in different colours) for $\mathrm{N}, \mathrm{M}_{\mathrm{i}}$ or M over a variable field of $\mathrm{w}_{0}$ and $\Psi$ for a specified joint rotation $\phi$ and a given normal force N (zero as default). This graphs then may serve for visual calculation checks, the discovery of iteration solutions (see next chapter) and to get a principle idea about the joint behaviour when varying deformation parameters.

The resolution of the graph depends on the value given as "Iteration mesh" (increasing mesh point number means increasing resolution). If the calculation time due to a high resolution is too long the "Stop" button can be used for an interruption. First of all the user will be asked for a graph type and the desired $\mathrm{w}_{0}$ - and $\Psi$-range to be displayed.

The following types of 3D-layer graphs are available (Fig.4-115):
type $0 \quad$ internal moment $M_{i}$ as third axis
type 1 difference between actual and given (Fig.4-114) normal force as third axis ( $\mathrm{dN}=\mathrm{N}_{\text {joint }}-\mathrm{N}_{\text {given }}$ )
For the graph types $0,1,6$ the colour of a rectangular graph element (resolution point) depends on the third axis value of its lower left corner. The colour palette covers the range from dark red over light red to light blue and dark blue indicating high positive, low positive, low negative and high negative values.
type 2 a combination of $M_{i}$ and $d N$ is drawn: for positive $d N$ values the corresponding $M_{i}$ values are drawn using the above given colour palette. In case of negative $d N$ a yellow colour is used in the case of a positive $M_{i}$ and a light yellow indicates a negative $M_{i}$ value.
type 3 analogous to type 2 exchanging $d N$ against $M_{i}$
type 4
type 5 analogous to type 2 and type 3, however those data areas are drawn in grey, where an "invalid force sign" has been identified
type 6 joint moment M as third axis
type 7 all areas where an "invalid force sign" is recognised are drawn in red, those where a "spring failure" occurs are drawn in blue and the remaining "regular" area is drawn in green.


Fig.4-115 3D layer-graph types

Fig.4-116 shows a comparison between the 3D layer graphs of CoBeJo-Joint and the corresponding 3D shape models drawn with the help of MS Excel.


Fig.4-116 3D diagrams - layer graphs and shape models

The maximum available $w_{0}$ - and $\Psi$-range is determined within reasonable limits:

- $w_{0}$ may vary between zero and $w_{0 \text { max }}$. For that purpose $F_{t, \max }$ is calculated as the sum of all strength values of the steelwork tension springs and the reinforcement springs

$$
\left.t, \max =\sum_{\mathrm{i}} \mathrm{~F}_{\max , \mathrm{C}_{\mathrm{i}}}+\sum_{\mathrm{k}} \min \left(\mathrm{~F}_{\max , \mathrm{E}_{\mathrm{k}}}, \mathrm{~F}_{\max , \mathrm{G}_{\mathrm{k}}}\right)\right)
$$

If this is higher than the maximum strength of spring $A\left(F_{t, \max }>F_{\max , A}\right)$ the spring $A$ itself probably will cause the joint failure and therefore $\mathrm{W}_{0 \max }=\mathrm{W}_{\max , \mathrm{A}}$ (to increase iteration speed a plastic strength plateau (if existing) of spring A can be reduced to only $5 \%$ if its length in view of $w$ ). If $F_{t, \max }<F_{\max , \mathrm{A}}$ the corresponding deformation of spring $A$ will be taken as $w_{0 \max }$ (see Fig.4-117). Finally the so-gained $w_{0 \text { max }}$ is multiplied with 1.05 to include also such iterative solutions where $\mathrm{w}_{0}=\mathrm{W}_{0 \text { max }}$.



Fig.4-117 Definition of $w_{0 \max }$

- $\Psi$ may vary between zero and $\phi$.

The user may decide either to display this maximum $w_{0}-$ and $\Psi$-range or to pick out a part of it by entering the graph limits.

As default graph type 4 (see Fig.4-118) will be displayed within the maximum $\mathrm{w}_{0}-$ and $\Psi$-range. This type of graph easily helps to identify iteration solutions (see 4.3.6.3). Such solutions are located at the intersection of red, blue and yellow colour (per definition above these are the points where $\mathrm{M}_{\mathrm{i}}=0$ and $\mathrm{dN}=0$, so fulfilling equilibrium). If $\phi$ and N input did not change since a previous iteration (4.3.6.3) the stored iteration results ( $\mathrm{w}_{0}-\Psi$ combinations) will be indicated in the graph with a light blue crosshair.


Fig.4-118 Graphical results
The following actions can be carried out with the help of the graphical result window: The "info" button gives an overview of the following possibilities. Moving the mouse pointer over the 3D layer graph will show the corresponding fundamental calculation results (see 4.3.6.1) as well as the deformed joint for the actual deformation state $w_{0}-\Psi-\phi$. A double click on the mouse even gives all details to the actual joint
deformation state (Fig.4-110). With a mouse click while pressing the shift-key a zoom window can be defined. The button "full" refreshes the original $w_{0}-\Psi$-range again. Specific $\mathrm{w}_{0}-\Psi$-points (max.20) can be accentuated on the graph with the "point" button. "Clear" erases all these points again. The graph type according to Fig.4-115 can be change with "Type".

Clicking on the right mouse button enables to pick a first and then a second point of a sectional cut through the 3D layer graph as shown in Fig.4-119. Instead of defining these cutting points visually they may also be defined by values with the help of an input box which opens when pressing shift and the right mouse button simultaneously. A single mouse click interrupts the definition of a second zoom window corner or a second cutting point.


Fig.4-119 Sectional view of 3D graph
Within the sectional graph the dN -course is drawn as a green polygon, the joint moment M is drawn in grey and the internal moment $\mathrm{M}_{\mathrm{i}}$ is drawn in red. The resolution of these curves depends again on the chosen "iteration mesh" in Fig.4-114. The red diagram background indicates the positive data area, the blue background indicates the negative one. All three polygons are drawn with a thin line if a spring failure or an invalid force sign has been observed, otherwise a thick line is used. The iteration (4.3.6.3) will determine such $w_{0}-\Psi-\phi$-combinations, where $M_{i}$ and $d N$ are zero simultaneously (successful equilibrium within the joint) as shown for the example calculation in Fig.4-119. Attention has to be paid if the curves are passing the zero level in a nearly perpendicular way, because this indicates a spring failure and therefore no real solution or equilibrium.
Again also with the sectional graph some mouse actions are possible: Moving the mouse pointer over the graph will show the corresponding (dashed line) fundamental calculation results (see 4.3.6.1) as well as the deformed joint for the actual deformation state $\mathrm{w}_{0}-\Psi-\phi$. A double click on the mouse even gives all details to the actual joint
deformation state (Fig.4-110). Clicking on the right mouse button will accentuate the actual point within the 3D layer graph (Fig.4-119-top), where a single mouse click leads back to the sectional view again. With "Done" in Fig.4-119-bottom the sectional graph is closed and the 3D layer graph displayed again (Fig.4-118).

With the button "Done" in Fig.4-118 the graphical presentation finally can be finished to start new calculations (Fig.4-104).

### 4.3.6.3 Fundamental iteration



Fig.4-120 Fundamental graph and iteration window

The complex fundamental iteration procedure itself is based on two sub-iterations which should be explained now as a starting point.

## $\mathrm{M}_{\mathrm{i}}=\mathbf{O}$ iteration ("iteration_psi")

For a given $\mathrm{w}_{0}$ and $\phi$ that special corresponding $\Psi$ will be determined (within the boundaries $\Psi_{\mathrm{a}}$ and $\Psi_{\mathrm{b}}$ ) where the internal moment $\mathrm{M}_{\mathrm{i}}$ is equal to zero (internal joint equilibrium). This iterative procedure is based on the bisection method described in Fig.4-16. So starting with the boundaries $\Psi_{a}$ and $\Psi_{b}$ and afterwards also for their bisection point the corresponding internal joint moments $M_{i}$ is calculated using the fundamental procedure "basic0" described in chapter 4.3.6.1. Knowing these three $\mathrm{M}_{\mathrm{i}}-$ values belonging to the three $\Psi$-values ( $\mathrm{w}_{0}$ and $\phi$ defined) the $\Psi$-boundaries are adapted so that the $\mathrm{M}_{\mathrm{i}}=0$ intersection again lies in between the new $\Psi$-boundaries. This halving of the $\Psi$-boundaries is repeated until one of the following criteria is fulfilled: Either $M_{i} \approx 0$ with a given accuracy ( $1 \mathrm{Nm} /$ precision factor=1) or the iteration time exceeds a stop limit ( 10 seconds) or the change of $\Psi$ lies below a stop limit ( 0.001 mrad ). If in the last case a less severe accuracy limit ( $100 \mathrm{Nm} /$ precision factor=1) is met this will be accepted anyway as a solution. Otherwise this indicates a drastic jump of $M_{i}$ due to a spring failure and therefore no solution to be found.

The course of $M_{i}$ depending on $\Psi$ for a defined $w_{0}$ and $\phi$ can easily be determined as a vertical sectional cut through the 3D layer graph.



Fig.4-121 Iteration example

## dN=0 iteration ("iteration_w0")

For a given $\phi$ that special corresponding $w_{0}$ (depending on $\Psi$ ) will be determined (within the boundaries $w_{0 a}$ and $w_{0 b}$ ) where the error of the normal force $d N=N_{j o i n t}-N_{\text {given }}$ is equal to zero (external joint equilibrium). While varying $w_{0}$ also $\Psi$ will be changed (within the boundaries $\Psi_{\mathrm{a}}$ and $\Psi_{\mathrm{b}}$ ) according to the procedure "iteration_psi" to fulfil the condition $\mathrm{M}_{\mathrm{i}}=0$ (internal joint equilibrium) simultaneously. So "iteration_w0" is an iteration loop covering "iteration_psi" as a sub-iteration.

First it will be checked for the given boundary $\mathrm{w}_{0 \mathrm{a}}$ if by applying "iteration_psi" at all any $\Psi$ can found fulfilling $M_{i}=0$. If yes then the corresponding $d N_{a}$ will be stored. Otherwise $\Psi_{a}$ is decreased in steps of $\left(\Psi_{b}-\Psi_{a}\right) / 10$ (minimum value is zero), $\Psi_{b}$ is increased in steps of $\left(\Psi_{b}-\Psi_{a}\right) / 10$ (maximum value is $\phi$ ) and $w_{0 a}$ is increased in steps of $\left(w_{o b}-w_{0 a}\right) /$ (iteration mesh). Then this check is repeated.

An analogous procedure is done for $w_{0 b}$ : It will be checked for the given boundary $w_{0 b}$ if by applying "iteration_psi" at all any $\Psi$ can found fulfilling $M_{i}=0$. If yes, then the corresponding $d N_{b}$ will be stored. Otherwise $\Psi_{a}$ is decreased in steps of $\left(\Psi_{b}-\Psi_{a}\right) / 10$ (minimum value is zero), $\Psi_{\mathrm{b}}$ is increased in steps of $\left(\Psi_{\mathrm{b}}-\Psi_{\mathrm{a}}\right) / 10$ (maximum value is $\phi$ ) and $w_{0 b}$ is decreased in steps of $\left(w_{0 b}-w_{0 a}\right) /$ (iteration mesh). Then this check is repeated.

Having found the corresponding dN -values $\left(\mathrm{dN}_{\mathrm{a}}\right.$ and $\left.\mathrm{dN} \mathrm{N}_{\mathrm{b}}\right)$ to the given $\mathrm{w}_{0}$-boundaries ( $\mathrm{w}_{0 \mathrm{a}}$ and $\mathrm{w}_{0 \mathrm{~b}}$ ) under the condition that $\mathrm{M}_{\mathrm{i}}=0$, the iteration can be started now applying the bisection method: For the bisection point $w_{0 a b}$ in between the actual boundaries $w_{0 a}$ and $\mathrm{w}_{0 \mathrm{~b}}$ that corresponding $\mathrm{dN}_{\mathrm{ab}}$ is determined with the help of the sub-iteration "iteration_psi", where $M_{i} \approx 0$. Knowing these three dN -values belonging to the three $\mathrm{w}_{0^{-}}$ values (respective $\Psi$-values to each $\mathrm{w}_{0}$; fixed $\phi$ ) the $\mathrm{w}_{0}$-boundaries are adapted so that the $\mathrm{dN}=0$ intersection again lies in between these new $\mathrm{w}_{0}$-boundaries. This halving of the boundaries is repeated until one of the following criteria is fulfilled: Either $\mathrm{dN} \approx 0$ with a given accuracy ( $5 \mathrm{~N} /$ precision factor=1) or the iteration time exceeds a stop limit ( 10 seconds) or the change of $\mathrm{w}_{0}$ lies below a stop limit $(0.01 \mu \mathrm{~m})$. If in the last case a less severe accuracy limit ( $500 \mathrm{~N} /$ precision factor=1) is met this will be accepted as a solution anyway. Otherwise this indicates a drastic jump of $d N$ due to a spring failure and therefore no solution to be found.

If "iteration_psi" could not find any $\Psi_{a b}$-solution that indicates that either a spring failure or even more than only one $\Psi$-solution fulfilling $M_{i}=0$ lies in between the mesh points $\Psi_{a}-\Psi_{b}$ (in the second case a mesh refinement is recommended). Such problems will be stated by a message box. To clarify such iteration problems the graphical tools provided in 4.3.6.2 proved to be really helpful.


Fig.4-122 Iteration example

## Fundamental iteration procedure

An iterative procedure to find all deformation states of the joint ( $w_{0}$ and $\Psi$ to a given ф) fulfilling internal and external equilibrium ( $\mathrm{M}_{\mathrm{i}}=0, \mathrm{dN}=0$ ) is launched pressing the button "iteration" in Fig.4-115. The iteration for $w_{0}-\Psi$-solutions to such a given $\phi$ is done in the following steps (a time consuming iteration may be interrupted by a "Stop" button).

## Calculation of data field

In a first step a mesh will be put over the maximum considerable range of $w_{0}$ and $\Psi$. The mesh width depends on the chosen value for the "iteration mesh" (Fig.4-115) defining the number of subdivisions for both axes $w_{0}$ and $\Psi$ (therefore doubling the iteration mesh factor quadruples the iteration time!). Then for each mesh node ( $\mathrm{w}_{0}, \Psi, \phi$ ) the check values $\mathrm{M}_{\mathrm{i}}$ and dN in view of a successful joint equilibrium are determined with the help of the fundamental joint calculation "basic0" (chapter 4.3.6.1). This procedure is quite similar to that for the fundamental graphs (4.3.6.2):

In an external loop $w_{0}$ will be varied between zero and $w_{0 \text { max }}$ (as defined in 4.3.6.2), an internal loop changes $\Psi$ between zero and $\phi$. To increase the iteration speed before starting any fundamental joint calculation it will be checked, if the actual combination of $w_{0}$ and $\Psi$ makes sense at all and if not, this mesh node gets the flag "no solution" immediately without further calculations. As illustrated in Fig.4-123 a wise lower limit of $\Psi$ for a given $w_{0}$ can be defined where at least the highest steelwork row is subjected to tension and not to compression. A wise upper limit of $\Psi$ for a given $w_{0}$ is reached, where all C -springs are ruptured.


Fig.4-123 Limits of $\Psi$ to a given $w_{0}$

Search for solutions
After such a data field of $M_{i}$ and $d N$ over $w_{0}$ and $\Psi$ has been calculated the $\mathrm{w}_{0}$ - and $\Psi$ loops are passed through again and for any mesh rectangle (Fig.4-124) it will be checked, if a solution within these mesh limits is possible at all. An indication for a
possible solution is that $M_{i}$ of at least one corner has a different sign than that at the other corners (for a boltless joint without any steelwork connection no separation bar 2 exists, therefore $\mathrm{M}_{\mathrm{i}}=0$ for any $\Psi$ and therefore this criterion will always be fulfilled). Simultaneously to the previous criterion also dN of at least one corner has to have a different sign than that at the other corners (or dN must be equal zero in at least one corner). For those mesh rectangles fulfilling both criteria a detailed search for a solution will be launched starting the procedure "iteration_w0" (described above). In the case that a solution really could be found and despite an eventual broadening of the $\Psi$ boundaries by "iteration_w0" this solution ( $\mathrm{w}_{0}, \Psi$ ) still lies in between the investigated mesh rectangle (otherwise this solution will be found again during the iteration for another rectangle and therefore should not be stored twice) the found values $w_{0}$ and $\Psi$ are stored as a solution. Then the search for other deformation states fulfilling equilibrium will be continued. Attention has to be paid to the fact that due to the described iteration method at maximum one solution per mesh rectangle will be found. So depending on the actual joint configuration the requirements on the iteration mesh vary. As a compromise between iteration time and reliability of the results an iteration mesh value between 50 and 100 is recommended. In special cases of critical points, where solutions are really close together the fundamental graphs (see chapter 4.3.6.2) using the zoom option are helpful.


Fig.4-124 Search for solutions within iteration mesh rectangles

## Presentation of solutions

At maximum 20 different solutions are stored during the search for solutions. A table summarising the gained $\mathrm{w}_{0}-\Psi$ combinations for the given $\phi$ and N will be displayed (see Fig.4-125). A single click on such a solution will copy the exact $w_{0}, \Psi, \phi$ values to the fundamental calculation window (Fig.4-106), a double click will directly start the fundamental calculation for this $\mathrm{w}_{0}-\Psi-\phi$ combination ending up with the detailed joint and component results (Fig.4-110). With the button "done" the solution table can be closed again to continue with new iterations, graphs or fundamental calculations.


Fig.4-125 Solution table from fundamental iteration

### 4.3.6.4 Summary

All these basic calculations evaluating the detailed joint response and the corresponding individual spring actions for a specific joint deformation state ( $\mathrm{w}_{0}, \Psi, \phi$ ) proofed to be a helpful tool during the development of the program in view of check calculations. Beyond that they can be used by an experienced user to get familiar with principles of joint detailing and failure modes. As the program has been built up starting with these basic calculations the explanation of the mechanical and mathematical background of joint iterations seemed to be much easier in that order. For the everyday use of the CoBeJo-Joint module the "basic calculations" will rarely be used directly, however they are the basis of the following main calculations.

### 4.3.7 Main joint calculations



Fig.4-126 Main joint calculation window (assembly0.frm)

The objective of the main joint calculation (Fig.4-126) is the derivation of an $\mathrm{M}-\phi$ curve representing the relation between the applied joint moment $M$ (hogging) and the corresponding joint rotation $\phi$. This is done by an assembly of all non-linear translational component springs considering post limit effects (also decreasing branches) and even failure of individual components. Obviously in view of such non-linearities the load history plays a significant role and therefore the calculation is performed rotationcontrolled gradually increasing the joint rotation $\phi$ step by step from zero to any upper limit. So within a loop increasing $\phi$ there will be a "fundamental iteration"(see 4.3.6.3) for each of these $\phi$-states.

### 4.3.7.1 Assembly boundaries



Fig.4-127 Definition of assembly boundaries

First some assembly boundaries have to be defined using the input tools collected in Fig.4-127:

- During the increase of joint rotation it may occur that some individual spring deformations are decreasing. The user has the opportunity to decide whether the corresponding spring force in such a case of component unloading should be calculated using the unloading stiffness ("Unloading linear", see Fig.4-108) or if the force should always be taken as lying on the w-F curve itself (Unloading "along curves", see Fig.4-107).
- The rotation limits can be defined as $\phi_{\text {start }}$ and $\phi_{\text {end }}$. In view of the iterative rotation increase from step to step (deformation history of all contributing components) in the case of a "linear unloading" $\phi_{\text {start }}$ has to be set zero. In case of an unloading "along curves" the iterations may even be started with any arbitrary value of $\phi_{\text {start. }}$ The number of $\phi$-states between $\phi_{\text {start }}$ and $\phi_{\text {end }}$ can be defined with the " $\phi$-step" input box. A higher number of $\phi$-states increases the resolution of the aimed $\mathrm{M}-\phi$ curve and therefore also its accuracy.
- For each $\phi$-state all possible solutions fulfilling joint equilibrium (see fundamental iteration procedure) will be stored. So for a high number of $\phi$-states also the number of solutions increases. A maximum number of solutions to be stored to an internal data field can be defined with "max.sol.nr.".
- The mesh width used for each "fundamental iteration" can be defined as "Iteration mesh" (see Fig.4-124).
- An acting normal force at the joint edge ("N") can be entered as well. However attention has to be paid that an existing normal force drastically restricts the valid $\phi$ range where the compression rows are still exposed to compression and the tension rows are still in tension. Whereas the influence of that effect has been taken into consideration within the Eurocodes [60] by a general upper normal force limit of $10 \%$ of the plastic beam resistance, those invalid joint deformation states with "invalid force signs" automatically will be high-lightened by CoBeJo-Joint for the actual joint configuration. Any normal force is assumed to act at the layer of the compression row ( $\mathrm{y}=0$ ) and therefore the joint moment also will be linked to that special point in case of $N \neq 0\left(M^{\prime} \neq M\right.$, see also Fig.4-44)!


### 4.3.7.2 Component assembly

Now the assembly of all translational spring curves can be launched with "Start iterations". Alternatively a recently performed assembly calculation may be loaded: Instead of the iterations described afterwards the same type of results can directly be taken from a previously saved output file (file type *.jo2; see 4.3.7.4 "joint curves") followed by the result presentation (Fig.4-130). Loading a result file requires that the actual joint configuration has already got a unique ID (joint identification, see 4.3.4) and that this agrees with the ID of the saved results to ensure coherence between the actual joint configuration and the results.

The assembly procedure is built up quite similar to the "fundamental iteration procedure" however with a further external loop changing $\phi$ within the limits $\phi_{\text {start }}$ and $\phi_{\text {end }}$ in steps of ( $\left.\phi_{\text {start }}-\phi_{\text {end }}\right) /(\phi$ steps). The iteration process is indicated by a flood box. A time consuming assembly may be interrupted with a "Stop" button.

For each $\phi$-state the following calculations are carried out (for further explanations the previous $\phi$-state will be called $\phi_{p}$, the actual one is $\phi_{\mathrm{a}}$ ):

- First a data field of $M_{i}$ and $d N$ values is calculated depending on the variables $w_{0}$ and $\Psi$.
- Then all possible solutions are searched. Apart from the corresponding spring deformations, spring forces, joint deformations and joint forces also the following data will be stored for each found solution:
Internal work ( $W_{i}$ ):
The work done by all the internal component forces is calculated to come from the relevant deformations related to the previous rotation state $\left(\phi_{p}\right)$ to those of the actual one ( $\phi_{\mathrm{a}}$ ), see Fig.4-128.


Fig.4-128 Examples for the work done by internal forces

## External work ( $W_{\mathrm{a}}$ ):

The work done by the external joint moment is calculated to come from the previous rotation state ( $\phi_{\mathrm{p}}$ ) to the actual one ( $\phi_{\mathrm{a}}$ ), see Fig.4-129. As the calculation of the $\mathrm{M}-\phi$ curve is done in $\phi$-steps the gained "curve" strictly is only a polygon and therefore the calculated external work to come from $\phi_{p}$ to $\phi_{a}$ is only an approximation by a linear interpolation. The deviation to the exact value
increases if one component undergoes a drastic change of stiffness within this $\phi$ step $\phi_{p}$ to $\phi_{a}$ (little deviation for higher number of " $\phi$-steps").

$$
W_{e}=\left|\frac{M_{p}+M_{a}}{2}\right| \cdot\left(\phi_{\mathrm{a}}-\phi_{\mathrm{p}}\right)
$$



Fig.4-129 Example for the work done by external moments

## Work difference ( $\left.\Delta W=/ W_{i}-W_{a}\right)$ ):

Finally the difference between internal and external work is determined as an indicator for the accuracy of a solution. For a perfectly small resolution of the M- $\phi$ curve (infinite number of $\phi$-steps) the work difference for the decisive solution would be zero.

- If only one solution could be found for the actual $\phi$-state no further investigations are necessary. Otherwise the one decisive solution has to be gained with the criterion that the work difference should be zero (or at least a minimum).
If already for an arbitrary starting rotation ( $\phi_{\mathrm{a}}=\phi_{\text {start }}$ ) more than one solution could be found the user will be asked to define one of the gained solutions as the decisive one (so this should already be known from a previous assembly starting with $\phi=0$ ).
- For the so-gained decisive solution the corresponding work difference will be converted into the following reference value, which afterwards will be displayed together with the results as an indicator for the reliability of the curves.

$$
\overline{\Delta W}=\frac{\Delta W}{\left|\frac{M_{p}+M_{a} \text { dececisive }}{2}\right|} \quad[\% / \mathrm{mrad}]
$$

- Finally also the corresponding translational spring deformations are stored for this decisive solution to serve as reference points $\left(w_{i}\left(\phi_{p}\right)\right)$ for the next $\phi$-state.

Now the above procedure is repeated for the next $\phi$-state.

### 4.3.7.3 Warnings

After the calculations have been finished for the last $\phi$-state ( $\phi_{\text {end }}$ ) it will be checked if the maximum observed work deviation $\overline{\Delta W}_{\text {max }}$ exceeds an info limit of $2 \% /$ mrad or even a warning limit of $10 \% / \mathrm{mrad}$. In such cases the following messages will be displayed:

- The difference between internal and external work is bigger than $2 \% / \mathrm{mrad}$. This indicates a deviation of the real $\mathrm{M}-\phi$ curve from its linear interpolation polygon
(stiffness change of spring). So you could try to reduce the $\phi$-intervals by increasing the number of $\phi$-steps to improve accuracy.
- The difference between internal and external work is bigger than $10 \% / \mathrm{mrad}$. This indicates a considerable deviation of the real M - $\phi$ curve from its linear interpolation polygon (remarkable stiffness change or even failure of one spring). So try to reduce the $\phi$-intervals by increasing the number of $\phi$-steps to get a more reliable solution. (In the case of a component failure $\overline{\Delta W}_{\max }>10 \% / \mathrm{mrad}$ commonly cannot be avoided and therefore this warning then may be ignored.)
Before disregarding such a warning it should be checked if a refinement of $\phi$-steps helps to improve accuracy by a closer approximation of the polygon to the real curve. The info and warning limits as well as the default data for the $\phi$-steps so far have defined based on the experience with a set of check calculations. Further comprehensive parameter studies using typical joint configurations should help to improve these values.

If some warnings have been observed for one of the several $\phi$-states during the procedures "iteration_w0" and "iteration_psi" due to more than one solution within an iteration rectangle or due to a spring failure they will now be displayed in a collected informative message box. In the case of multi-solutions a more dense iteration mesh is recommended (increase "Iteration mesh"; attention: doubling the mesh factor quadruples the iteration time!). If the reason really is a spring failure this warning can be ignored. Such a decision can easily be taken from the "basic joint calculations" (see 4.3.6.2).

### 4.3.7.4 Presentation of results

At the end the calculation window (Fig.4-126) will be modified for the presentation of the results as shown in Fig.4-130. To come back to the definition of assembly boundaries (Fig.4-127) for a new calculation the button "Done" has to be pressed.


Fig.4-130 Presentation of main calculation results

The several presentation tools should now be described in detail to take maximum profit of the results: Fig.4-131 shows the joint curve window, Fig.4-135 shows the joint deformation window and Fig.4-136 shows the component response window (all these three being part of the overall presentation window shown in Fig.4-130).

## Joint curves



Fig.4-131 Joint curve window
At the top of this window the chosen assembly boundaries such as the normal force, the iteration mesh, the $\phi$-range including the number of steps and the type of unloading (linear or along the curve) are stated, followed by the maximum deviation values in view of the normal force, the internal joint moment and the work difference considering all solutions for the whole $\phi$-range.

The core of Fig.4-131 is represented by the joint curves themselves. For both of the two axes list boxes enable to select between the joint rotation $\phi$ [mrad], the joint moment $-\mathrm{M}[\mathrm{kNm}]$ (hogging positive), the compression dent $\mathrm{w}_{0}$ [mm] and the rotation of the loadintroduction separation bar $\Psi$ [mrad]. The variety of different axes combinations helps to get an immediate idea about the joint response. All decisive solutions are indicated with thick circles, the other (non-decisive) solutions are drawn as small points. The colour of these circles or points represents the corresponding joint state: black is normal, blue indicates a spring failure and red indicates an invalid force within a component. If $\Delta \mathrm{W}_{\max }<2 \% /$ mrad the polygon connecting the decisive solution circles is draw with a thick line, if $\overline{\mathrm{W}}_{\text {max }}>2 \% /$ mrad the polygon is a thin line and if $\overline{\Delta W}_{\max }>10 \% / \mathrm{mrad}$ the polygon is a dashed line as a kind of warning in view of reliability. For each decisive solution (circles) the corresponding work difference $\overline{\Delta W}$ is indicated by a vertical white bar; the longer this bar the higher is this deviation. This gives an idea within which $\phi$-range a refinement of the curve would be recommendable.

With the option buttons "full info" and "curve only" it can be decided whether all the mentioned additional information should be given in the graphs or if it should be focused on the bare curve itself. "Load curve" enables to give an additional reference curve drawn in light green for a direct comparison. Such a reference curve can be loaded from a *.spr-file (Fig.4-90). The values stored in the first column are taken for
the horizontal axis, those of the second column are taken for the vertical axis. The reference curve can be hidden or shown again with the buttons "Hide curve" or "Show curve". Analogously an actual solution curve (polygon) can be saved to such a *.spr file for a later comparison with other calculations or for an export to other programs. The name of the curve will automatically be set together by the joint file name and the actual axes. The first column is filled with the values of the horizontal axis, the second column stores the vertical axis. To refine an M - $\phi$ curve only in such $\phi$-ranges where the observed work deviation has been relatively high the actual $M-\phi$ solutions may be added to an already saved $\mathrm{M}-\phi$ curve using the button "Add curve to file". This might help to save iteration time, however generally it is recommended to start a completely new joint calculation just with a higher number of $\phi$-steps but covering again the full $\phi$-range.

If the MS Windows clipboard has been used in the meantime it may be necessary to "refresh" the graph. With the "+" button followed by a mouse click definition of two points within the graph a zoomed view can be realised. The "-" button restores the graph within the maximum axes boundaries of all found solutions. Pressing "Shift" and the "-" button simultaneously sets the axes limits to those of the actually loaded reference curve.

With the button "Table" it can be switched from the graphs to a table collecting all solution data in view of the corresponding $\phi, M, w_{0}$ and $\Psi$ values (see Fig.4-131, right side). "Curve" changes back to the graphs. A double click with the mouse on one of the solutions either in the graph or the table also cause a switch between graphical and tabular result representation. The double clicked solution will be high-lightened.

Moving the mouse over the graph (mouse pointer is a crosshair) gives the axis values of the actual mouse position next to the axes list boxes. However one of the most interesting tools certainly is the possibility to see the direct coherence between a solution point and both the corresponding joint deformation as well as the corresponding component's response. So when moving the mouse over the graph whilst the left mouse button is kept pressed (or clicking on a solution in the table) will cause the following procedure: the closest decisive solution point to the actual mouse position is evaluated, high-lightened by a big circle and the corresponding data such as $\phi, \mathrm{M}, \mathrm{w}_{0}, \Psi$ and all corresponding deviation values are displayed within the lower part of the joint curve window (Fig.4-131). Furthermore the corresponding joint deformation state (Fig.4-135) is updated as well as the corresponding component responses (Fig.4-136). If one of the axis is defined to display the joint rotation $\phi$ the mouse pointer becomes a horizontal or vertical arrow if the mouse button is pressed, indicating that only that mouse movement in the direction of the $\phi$-axis will be considered for the search of the closest decisive solution (this eases to follow the full load history from $\phi_{\text {start }}$ to $\phi_{\text {end }}$ without any unintended jumps). Otherwise the mouse pointer remains a crosshair and both the actual horizontal and vertical mouse position are of interest for the search of the closest decisive solution. When pressing "Shift" additionally (plus left mouse button, plus mouse move) not only the closest decisive but any closest solution can be caught (however those non-decisive solutions normally are not of further interest). Fig.4-132 shows a kind of film representing the graphs resulting from a mouse move along the $\phi-\mathrm{M}$ curve.


Fig.4-132 Corresponding joint deformation to respective $\phi$-M points

Finally the overall gained assembly results can be saved to an output file with the help of the button "Save". First the file name has to be defined with the file management window (Fig.4-86). If the actual joint configuration has ever been saved and therefore has already got its unique joint-ID (see 4.3.4), which enables a clear assignment between joint input and corresponding joint output file, it can be decided between three different output types (if no joint-ID is available type 0 will be chosen automatically):

- type $0=$ SHORT file version (*.jo1): As shown on page 306 only a very short file will be produced containing $\phi, M, w_{0}$ and $\Psi$ for the decisive solutions only. Such an output file is intended for a data transfer to other applications but cannot be loaded later instead of carrying out the same assembly iterations again (too little data available).
- type $1=$ LONG file version (*.jo2): As shown on page 307 an extended file will be produced containing $\phi, M, W_{0}$ and $\Psi$ as well as all provisional component results (corresponding $W-F$ points and force degrees) however only for the decisive solutions. Such a result file can be loaded later instead of carrying out the same iterations again (see 4.3.7.2).
- type $2=$ FULL file version (*.jo2): As shown on page 308 a complete file will be produced containing $\phi, M, w_{0}$ and $\Psi$ as well as all provisional component results (corresponding w-F points and force degrees) for all solutions. Also such a result file can be loaded later instead of carrying out the same iterations again (see 4.3.7.2).
Together with MS Excel these output files e.g. may be used to draw relationships such as between $\phi$ and the corresponding component deformations $w_{i}$ or component forces $F_{i}$ (examples are illustrated in Fig.4-133 and Fig.4-134).


Fig.4-133 Coherence example $\phi$ - $w_{i}$ gained from saved output file


Fig.4-134 Coherence example $\phi$-Fig gained from saved output file

## SHORT result file version: DECISIVE solutions only








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## Joint deformation



Fig.4-135 Joint deformation window
The graphical presentation of the actual joint deformation state has already been described in detail in chapter 4.3.6.1 (presentation of results, Fig.4-112). It comprises the actual component deformations including the force degrees (incl. eventual spring failures) and the rotation centres of the separation bars.

Moving the mouse over the graph displays the actual geometrical values $x$ and $y$ below the graph. The shear panel size, the connection size, the deformation factor and the joint scale may be varied using the relevant scroll bars. The presentation of the force degrees can be manipulated with option buttons: either "dark" or "light" bars are indicating the actual force degree (percentage of use) for each component or this additional information can be switched off.

## Component response



Fig.4-136 Component response window

The w-F values corresponding to the presently chosen solution is drawn for each contributing component as a red point within the relevant w-F curve. Due to an eventual linear unloading this point must not always lie on the curve itself. By clicking on the table of contributing components one being of present interest can be chosen. The state of the option buttons "same scale" or "max. scale" decides whether all component curves are drawn with the same axes limits or if for each one the scale should be stretched to fill the full diagram area.

### 4.3.7.5 Final comparisons

Fig.4-137 shows a comparison between the results gained for the described example joint ("thesis.jip", see page 263) and those results from the same joint, however where only the layer of reinforcement in tension has been removed. So this is a comparison between a composite joint and its steelwork counterpart (final state $\leftrightarrow$ erection state). The gain of stiffness and resistance by adding and activating the slab reinforcement for the joint action is obvious. In this composite joint example ("thesis.jip") the ductility of the reinforcement deliberately has been reduced just to demonstrate a spring failure. However in normal practice the composite joint will provide a clear moment plateau for plastic redistribution (thick line in Fig.4-137 with modified spring E with 8 mm ductility instead of 4 mm ).


Fig.4-137 Composite joint compared to bare steel joint counterpart

The difference in view of the $M-\phi$ curve once calculated with a linear component unloading and once with an unloading along the F-w curve can be seen in Fig.4-138. As for the example joint "thesis.jip" only the component A will be unloaded the difference in the resulting curves is not really important.


Fig.4-138 Different $M$ - $\phi$ curves due to different component unloading types

Finally a comparison should be performed between the sophisticated curve gained with the help of CoBeJo and the joint's key values (initial stiffness $\mathrm{S}_{\text {ini }}$ and moment resistance $\mathrm{M}_{\mathrm{R}}$ ) according to the Eurocode [49,60] assembly procedures. The ECcalculation for the example joint "thesis.jip" is given in Fig.4-139. A comparative graph can be seen in Fig.4-140.
c ... spring stiffness $[\mathrm{kN} / \mathrm{mm}], F \ldots$ spring resistance $[\mathrm{kN}]$

$$
\begin{aligned}
& \text { compression row } \mathrm{A}+\mathrm{B} \text { : } \\
& C_{A B}=\frac{1}{\frac{1}{1000}+\frac{1}{\infty}}=1000 \quad F_{A B}=\min (300 ; 200)=200 \\
& \text { steelwork tension row } C+D \quad(y=100 \mathrm{~mm}) \text { : } \\
& C_{C D}=\frac{1}{\frac{1}{850}+\frac{1}{900}}=437 \quad F_{C D}=\min (130 ; 140)=130 \\
& \text { reinforcement row } E / 2+G \quad(y=150 \mathrm{~mm}) \text { : } \\
& C_{E G}=\frac{1}{\frac{1}{1}+\frac{1}{800}}=343 \quad F_{E G}=\min (50 ; 60)=50 \\
& \left.z_{\text {eq }}=\frac{437 \cdot 100^{2}+343 \cdot 150^{2}}{437 \cdot 100+343 \cdot 150}=127 \mathrm{~mm} \quad \text { [see EC3 }- \text { Annex } \mathrm{J}(\mathrm{~J} .38)\right] \\
& c_{\text {eq }}=\frac{437 \cdot 100+343 \cdot 150}{127}=749 \mathrm{kN} / \mathrm{mm} \quad[\text { see EC3 }- \text { Annex J (J.36)] } \\
& \mathrm{S}_{\text {ini }}=\frac{127^{2}}{\frac{1}{\frac{1}{1000}+\frac{1}{749}}}=6910 \mathrm{kNm} \\
& \mathbf{M}_{\mathbf{R}}=50 \cdot 150+130 \cdot 100=\mathbf{2 0 . 5} \mathbf{k N m} \quad(50+130<200 \quad \text { ok })
\end{aligned}
$$

Fig.4-139 Main joint characteristics calculated according to the Eurocodes [49,60]


Fig.4-140 Graphical comparison between sophisticated and simplified spring assembly results

### 4.3.7.6 Unbalanced joint loading

The present program version only enables the assembly for balanced joint moments ( $\beta=0$, see Fig.4-141). In the future this will be extended to fully unbalanced joints ( $\beta=1$, single-sided) including the behaviour of the components $F$ and $H$ as well as the frame spring SYS. Finally it will be enlarged to $0<\beta<1$.


Fig.4-141 Ratio of moment imbalance

For $\beta=0$ only three ( $\mathrm{w}_{0 \mathrm{r}}, \Psi_{\mathrm{r}}, \phi_{\mathrm{r}}$ ) of the overall seven ( $\mathrm{w}_{0_{\ell}}, \Psi_{\ell}, \phi_{\ell}, \mathrm{w}_{0 \mathrm{r}}, \Psi_{\mathrm{r}}, \phi_{\mathrm{r}}, \phi_{\mathrm{s}}$ ) degrees of freedom (Fig.4-142) had to be varied within the iterations, because due to symmetry the deformations on the left and right hand side are equal and the shear deformation even is zero. However the general structure of the program has already been built up in view of such an extension to unbalanced joint loading.


Fig.4-142 Joint degrees of freedom and remaining variables

### 4.3.8 Conclusions

Both the theoretical background as well as the handling of the CoBeJo-module "Joint characterisation" have been described. This program represents a powerful tool for the non-linear assembly of component springs according to the Innsbruck component model. As the pre- and post-processing possibilities have especially been adopted to the requirements of joint calculations it is a real alternative to conventional FEprograms, which have been used for check calculations [137]. In the present version, where the component characteristics have to be taken from tests or mechanical models this program is primary intended to researchers and advanced practitioners. Integrating this sophisticated assembly procedure into an overall joint characterisation package like the Module Bank Program [122] could increase its common use drastically.

Using comprehensive parameter studies the assembly procedure CoBeJo-Joint will now serve to investigate the applicability of the simplified Eurocode component assembly as well as the influence of overstrength effects on failure modes and joint rotation capacity.

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